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Exploring Conceptualisations of the Definite Integral in Mathematics Education: A Systematic Literature Analysis

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Abstract

Background/purpose. The teaching of integral calculus has become increasingly important in higher education due to its role in the development of advanced mathematical thinking. In this context, the definite integral is recognised as a complex mathematical object with multiple interpretations. The present article aims to systematically review the literature on its conceptualisations in mathematics education research, focusing on identifying the predominant approaches, uncovering theoretical gaps, and identifying opportunities to improve teaching and learning practices for this mathematical object.

Materials/methods. A systematic review of the literature was conducted between 2010 and 2024, following PRISMA guidelines. The search was conducted across databases including Scopus, ERIC, ScienceDirect, Taylor & Francis, and Google Scholar. In total, 64 articles were selected for analysis. Artificial intelligence tools and VOSviewer-based bibliometric network analysis were applied to identify conceptual patterns.

Results. Eight main conceptualisations of the definite integral were identified: area under the curve, antiderivative, accumulation, accumulation function, average value, numerical approximation, application to real situations, and solids of revolution. The results reveal a fragmentation between formal, technological, and curricular approaches. Bibliometric networks show poorly connected clusters, which indicates a weak link between theoretical frameworks and their implementation in teaching.

Conclusion. This study demonstrates that fostering a deep understanding of definite integrals requires integrated teaching, explicit attention to infinity, and thoughtful use of technology. Although this review provides an updated theoretical basis, it has limitations: it excludes studies prior to 2010 and focuses mainly on publications in English, thereby limiting the diversity of perspectives. It is suggested that future research expand the analysis to other languages and different educational levels.

1. Introduction

Throughout the history of mathematics, its development has enabled progress in various disciplines. The definite integral, in particular, has played a crucial role in the understanding of fundamental concepts such as area (Jones, 2015; Alves et al., 2019; Pambudi, 2022), volume, and the accumulation of quantities (Kouropatov & Dreyfus, 2014). This mathematical concept is not only essential in calculus, but also in fields as varied as physics (Sealey, 2014), engineering (Brijlall & Ndlazi, 2019), and economics.

Some researchers have addressed the challenges associated with teaching and learning the integral (Jones, 2018; Ely, 2017; Serhan, 2015; Huang, 2012). They highlight that one of the most recurrent problems when addressing this mathematical concept is the procedural knowledge that students and teachers have. This is because students perform better at calculating definite integrals than in "connecting the different representations of this" (Serhan, 2015; Mallet, 2013).

In this regard, recent research highlights that two important skills in integral calculus that can help with this problem are the ability to interpret and model using derivatives and definite integrals (Ely, 2017; Thompson & Silverman, 2008). Therefore, it is essential to develop teaching strategies that strengthen these skills and foster a deep understanding of the relationships among the various meanings of these mathematical objects and their applications in real-life contexts.

Given the above and considering the growing importance of teaching and learning the definite integral, it is essential that teachers, students, and researchers clearly understand what this mathematical object is. In particular, they need to be able to identify the different conceptualisations of the definite integral, as this promotes cognitive development and facilitates progressive mastery of the conceptual fields involved (Vergnaud, 2013).

Likewise, for researchers in educational mathematics, understanding how the definite integral has been conceptualised in the specialised literature is key to identifying prevailing approaches, detecting research gaps, and establishing which perspectives should be promoted or strengthened in educational practice. This need becomes even more important when you consider the challenges both students and teachers face in navigating the different meanings associated with this mathematical object.

In addition, conceptual and procedural errors often arise when dealing with definite integrals. Among the most frequent are: confusion between mental images of definite integrals and indefinite integrals, which leads to an ambiguous interpretation of this mathematical object and its properties (Jones, 2013); incorrect use of integration limits, either by reversing their order, misinterpreting their role within the interval, or assigning them mechanically without an understanding of their contextual meaning (Ely, 2017); as well as the difficulty in linking the calculation of areas with the notion of accumulation and with Riemann sums, which highlights a disconnect between the approximation process and its mathematical formalisation (Borji & Martínez-Planell, 2023; Sealey, 2014).

These errors not only reflect obstacles to learning integral calculus but also limitations in the conceptual frameworks in which the definite integral is taught and researched. Although the literature reports multiple meanings associated with this concept, such as the area under the curve (Jones, 2015) and the accumulation function (Aranda & Callejo, 2020; Kouropatov & Dreyfus, 2014; Martínez-Miraval & García-Rodríguez, 2023), among others. Few studies systematically review the different conceptualisations present in current research or analyse how these conceptualisations relate to or are articulated in teaching practices. This gap hinders the development of inclusive teaching proposals and coherent conceptual progressions.

Similarly, there is a lack of studies on the role of technology in learning definite integrals (Hamid et al., 2025). Although it is recognised that tools such as GeoGebra, Excel, or symbolic calculation

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software can enrich students' experiences, few studies systematically analyse how these mediations influence the construction of meaning. This gap results in a lack of specific guidelines for designing tasks or teaching units, limiting the transfer of research into teaching practice.

Furthermore, there is a lack of longitudinal studies that track students' development from an intuitive understanding of the subject and accumulation to formal definitions involving Riemann sums, and then to the Fundamental Theorem of Calculus and the accumulation function. This lack of research makes it difficult to track the evolution of mental structures over time (Greefrath et al., 2021). Another under-explored area is the analysis of how the definite integral is approached at different educational levels, as most studies focus on university contexts, leaving upper secondary education largely neglected. However, the conceptualisation patterns identified in higher education can offer valuable insights for rethinking and restructuring its teaching in earlier school grades.

Therefore, to promote meaningful learning of this mathematical object, this research contributes to identifying the conceptualisations prioritised in university classrooms. In this sense, its main contribution is to articulate the different perspectives—conceptual and technological—on the definite integral, thereby promoting a robust and transferable understanding of this concept across various educational contexts.

This articulation is fundamental because the study of definite integrals cannot be reduced solely to the memorisation of calculation techniques (Jones, 2018); on the contrary, it requires the mobilisation of different meanings that allow us to establish networks of connection and formalisation through conceptual definitions, the coordination of multiple registers of representation, and the incorporation of technological resources that enhance understanding. Precisely for this reason, it is also necessary to recognise and address the alternative conceptions that emerge among students (Garcia-Garcia et al., 2025), to guide, from a pedagogical and didactic perspective, strategies that help overcome the epistemic obstacles that persist in advanced stages of mathematical education.

At the same time, this research is being conducted to facilitate the integration of the different cognitive configurations developed by students and teachers, thereby contributing to a solid, coherent, and consistent didactic transposition of the definite integral.

Based on these considerations, the following research questions arise:

- What conceptualisations of the definite integral are evident in the review of academic literature?
- What technological resources are used in university classrooms to address conceptualisations of the definite integral?

2. Literature Review

Since the 1980s, the teaching and learning of definite integrals has been a field of research in various countries, including the United States, Colombia, Mexico, and Turkey, among others (Cordero, 2003; Aldana et al., 2020). These studies have explored the multiple meanings attributed to this mathematical object, as well as the difficulties and errors encountered by students in different university programmes and degree courses (Boigues et al., 2010; López-Leyton et al., 2024). Similarly, research has been conducted from a curricular perspective, examining the conceptual definitions proposed in textbooks and considered by teachers in the design of instructional materials (Hong, 2023b; Sevimli, 2016).

Although in the case of the derivative understood as a slope, the literature has explicitly documented various conceptualisations (Stanton & Moore-Russo, 2012; Nagle & Moore-Russo, 2014), much of the research on the definite integral has been more oriented towards the study of

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mental models, alternative conceptions, or personal and institutional meanings associated with this concept. In some cases, conceptualisations have been addressed in isolation, without a systematic analysis that considers the full set of possible typologies. Consequently, a review of the literature reveals a significant gap in research that comprehensively and specifically analyses these conceptualisations.

This gap justifies the relevance of the present study, whose purpose, apart from identifying them, is to explore them in greater depth and contribute theoretical and didactic elements that enrich educational and cognitive practices in the university setting.

3. Methodology

In order to answer the research questions of this article, a systematic literature review is conducted. This involves a methodical search that comprehensively synthesises and analyses the available information on a specific topic (Page et al., 2021; Moher et al., 2009). The information provided by systematic reviews enables stakeholders, teachers, and researchers. First, to have a solid understanding of the existing gaps in a field of research (Khan, 2020).

Second, to make decisions for planning future studies to close gaps based on the evidence gathered (Petticrew & Roberts, 2006).

The methodology used in this review was based on the PRISMA (Preferred Reporting Items for Systematic reviews and Meta-Analyses) guidelines (Page et al., 2021; Moher et al., 2009). This is to assemble an evidence base from relevant studies. To screen the research articles, a search was conducted in various databases, including Google Scholar, Scopus, ScienceDirect, Taylor and Francis, and ERIC. In addition, the impact h-index, measured by Scimago Journal & Country Rank (SJR), was considered.

In addition, the bibliographic manager Mendeley (Reis et al., 2022) and the software VOSviewer v.1.6.20 were used for network-based bibliometric analysis. This software facilitates the identification of clusters (Van Eck & Waltman, 2010). In other words, it allows the identification of correlations by grouping elements into clusters. Complementarily, the bibliometric characteristics of the research, such as citations, authors, journals, and years, were analysed using the Python Crossref REST API 1.5.0 (Ariza & Olatunde-Aiyedun, 2024; Lammey, 2016).

However, in relation to the methodological phases implemented. To guarantee a rigorous process of systematic review. It begins with the first phase, which consists of identification. In this stage, a search is conducted according to a protocol (Gutiérrez et al., 2025; Montiel et al., 2020) to identify articles on the teaching or learning of the definite integral. In which the research questions and the purpose of the study are taken into account. This search focused on specialised journals and specific databases. A total of 222 articles were obtained.

Subsequently, we proceed with selecting the researchers based on the protocol's implementation. The articles are manually selected using Mendeley, and the following inclusion and exclusion criteria are considered.

- The article dealt with the definite integral. Those related to applied mathematics, published in conferences, proceedings, or master's or doctoral theses, were excluded.

- The article dealt with digital tools used in the teaching or learning of calculus. Articles dealing with function, limit, and derivative were excluded. Likewise, those in which digital tools were used in the methodological part to analyse the results obtained by teachers or students, rather than as a means or artefact for the design or implementation of tasks, were discarded.

- The article addressed a theoretical or conceptual framework of Educational Mathematics. Those dealing with a mathematical object other than the definite integral were excluded.

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- The review only considered articles published in English. Those written in other languages, such as French or Mandarin, were excluded. If any articles in Spanish were found to be relevant, the first filter was to check whether the journal was indexed in Scimago. After reviewing its content, a decision was made on whether to include or exclude it.

At this stage, both bibliometric and content-related variables of interest were considered. This systematised selection yielded 91 articles relevant to the study. For the eligibility process, the principal investigator read the abstract, research question, results, and conclusions of the 91 articles. In addition, Mendeley was used to discard duplicate articles. This resulted in 64 research articles.

Finally, in the inclusion stage, the data from these 64 studies were extracted. Initially, the information was organised in a data matrix, and subsequently, the different conceptualisations found in the teaching and learning of the definite integral were qualitatively identified, along with the mathematical tasks associated with these conceptualisations and the possible technological tools used in them.

This entire process was organised and represented using a flowchart that provides a schematic overview of the analysis's different phases. This is shown in Figure 1, which illustrates the stages involved, from the identification of studies to the organisation and interpretation of the extracted information.

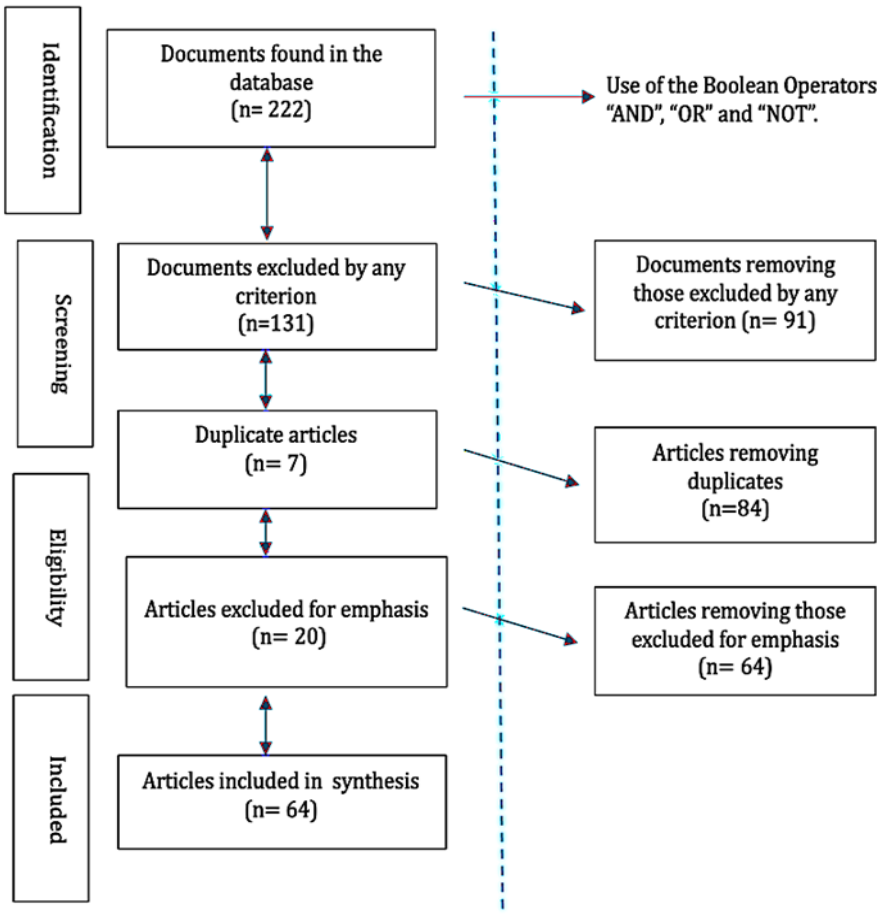

Figure 1. PRISMA flow chart adapted to the research

3.1. Identification: Search procedure

In the first methodological phase, a search process is initiated that follows an established protocol (Montiel et al., 2020). This protocol consists of two elements: an introduction, which includes the research question and the study's objective or purpose. A methodology made up of the bibliometric variables contemplated for the systematic review.

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These variables include aspects such as the title (Conceptual Understanding of Definite Integral with GeoGebra, Understanding the integral: Students' symbolic forms, Developing prospective mathematics teachers in Mexico: a lesson on the relationship between integration and differentiation), the period of the study (2010-2024), the language (English or Spanish), the type of publication (article in a high-impact indexed journal), the sources of information or databases (Google Scholar, ScienceDirect, Scopus and Taylor and Francis), the name of the journal (Journal of Mathematical Behavior, Educational Studies in Mathematics, ZDM – International Journal on Mathematics Education, EURASIA Journal of Mathematics Science and Technology Education).

Likewise, the H-index (journals found in Scimago), the main authors of the research (Michael Oehrtman, Patrick W. Thompson, Aneshkumar Maharaj), and the country in which the research is carried out (Tel Aviv University - Israel, Taiwan, Arizona, Mexico, among others).

This methodology also includes enquiry strategies, which involve search equations (Aromataris & Riitano, 2014). These strategies are based on search terms, such as keywords related to the research topic. These terms may include main concepts such as “definite integral” and “mathematics education,” as well as acronyms and synonyms. In addition, Boolean operators such as “AND”, “OR”, and “NOT” (Beirat et al., 2025; Abdul et al., 2022) are used to combine search terms and define the relationship between them. For example, the “AND” operator is used to search for articles containing all the specified terms, such as “definite integral” AND “mathematics education”. In contrast, the “OR” operator is used to search for articles containing at least one of the specified terms, while “NOT” is used to exclude specific terms from the search.

Similarly, truncation-wildcards and grouping of terms are used to structure search equations (see Table 1). Truncation and wildcards are symbols such as “”, “?” or “$” that are used to retrieve variations of a word. For example, “defini integral” would retrieve results containing “definite integra”, “integral”. The grouping of terms is done using parentheses, which allow you to group search terms and define the order of Boolean operations.

Table 1. Description of some of the search equations for the systematic review of the research

Database	Description
Scopus	"definite integral " AND "mathematics education "
	TITLE-ABS-KEY ("definite integral" AND "calculus" AND "education")
	"mathematics education " OR "technology" AND "definite integral"
ERIC	"definite integral" AND "technology"
google Scholar	("definite integral" AND ("calculus" OR "technology")

Finally, this methodology addresses the content variables of interest. These variables are the characteristics, attributes that a researcher wishes to analyse (Popay et al., 2006). In this study, the following are considered: the mathematical object, which includes the definitions and conceptualisations of the definite integral (such as area under the curve, accumulation, accumulation function); the theoretical framework (APOS, mathematical connections, semiotic representation registers).

Also, the type of methodology (qualitative, quantitative, mixed); the digital tools implemented (GeoGebra, Excel, Wolfram Alpha, Matlab); the field of educational mathematics in which each of the research projects is developed (contextualised to real phenomena or strictly mathematical); as well as the main ideas and some of the results obtained from each of the articles.

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3.2. Selection

The selection process for the 222 papers began with the application of the search strategies and the inclusion and exclusion criteria described above. First, the documents were added to Mendeley using the Research Information Systems extension.

Second, missing information was completed for each document, including the DOI, title, authors, and other fields. Third, documents were excluded based on some criteria. For example, documents that were not published in high-impact, indexed journals. Obtaining n=91 researches. Fourth, duplicate articles were eliminated. The final result of this process was n=84 research papers.

3.3. Eligibility

At this stage, the 84 articles were read by the principal investigator, who analysed their titles, abstracts, methodologies, and results. To identify whether they were in the context of the systematic review of this study. In addition, for each of these articles, artificial intelligence was applied to corroborate the principal investigator's findings. And thus, synthesise it in a data matrix in Excel clearly and concisely.

After reading each research paper in detail and discussing its content with the researchers of this study, 20 articles were excluded due to their emphasis. That is to say, when reading in depth, it was found that they were related to the topic of the definite integral in the field of educational mathematics. However, it did not directly address the interest of our review. This left a total of 64 articles, distributed as follows: Google Scholar = 10 articles; ScienceDirect = 10 articles; Scopus = 17 articles; Taylor and Francis = 20 articles; and ERIC = 7 articles.

3.4. Inclusion

Once the studies to be included in the final review for data extraction have been determined, an Excel matrix is prepared as mentioned above. Python Crossref REST API 1.5.0 is also used to extract bibliometric information, including citations, authors, publishers, and publication years. The purpose is to support the information obtained with VOSviewer v. 1.6.20 and to analyse bibliometric variables, such as content qualitatively. In addition, to identify, through clustering and visual maps, the keywords related to the conceptualisations of the definite integral during the period between 2010 and 2024.

3.5. Data Extraction and Analysis of Studies

In order to find the various conceptualisations of the definite integral in the research on Educational Mathematics, the theoretical background as well as the results of each article were read. The analysis of the data obtained from the studies was carried out using an inductive approach, as detailed below.

First step: to investigate what conceptualisations exist about the definite integral. First, we analysed its mathematical definitions, applications in different contexts (biological, physical, among others), associated semiotic representations, and connections with other mathematical concepts.

Second step: In this step, we looked for whether the articles referred to any specific theory to describe the conceptualisation of the definite integral. According to the methodology's criteria, articles that did not mention a specific theory were excluded from the analysis. Finally, the resulting 64 articles were classified according to the technological tools used in the proposed tasks and their functionality to facilitate the conceptualisation of the definite integral.

4. Results

The results obtained regarding conceptualisations of the definite integral include the definitions provided by academics, the applications chosen for the tasks, the semiotic

representations that emerged during the resolution, and the connections with other mathematical concepts. We present these results from a general perspective on the categorisations of conceptualisations, detailing the elements mentioned above.

It is important to note that each of these conceptualisations not only provides a unique perspective on this concept but also interrelates different branches of mathematics. This interconnection not only enriches understanding of the definite integral but also promotes metacognition, allowing both students and teachers to reflect on their own learning processes.

4.1. Conceptualisations of the definite integral in academic literature

The term "definite integral" has been interpreted by several authors (Kontorovich & Locke, 2023; Pambudi, 2022; Burgos et al., 2021; Mateus-Nieves & Font Moll, 2021; Greefrath et al., 2021; Alves et al., 2019; Ting, 2017; Tatar & Zengin, 2016; Serhan, 2015; Milovanović et al., 2011) mainly as the "area under a curve" in the context of calculus. This geometric interpretation is closely linked to the geometric concepts of perimeter and area, as it allows the area of a region in the Cartesian plane to be calculated. Furthermore, it is fundamental because it not only facilitates geometric understanding, but also the analysis of the definite integral (Jones, 2015), by visualising it as the process of adding infinitesimal areas.

In other words, the geometric conceptualisation of the definite integral, seen as the area under the curve, is directly related to Riemann sums (Mateus-Nieves & Font Moll, 2021), since they are an essential tool for estimating area by summing inscribed or circumscribed rectangles. These Riemann sums are intrinsically related to the concept of limit (Sealey, 2014), because by increasing the number of subintervals and making their length smaller, tending to zero, the value of the definite integral is obtained.

Thus, the definite integral has evolved beyond its conceptualisation as the area under the curve, expanding to other conceptualisations such as the calculation of volumes, length of a curve, accumulation, among others (Huffman Hayes, 2024; Gulkilik, 2022; Ergene & Özdemir, 2021; Benacka, 2016). In this sense, its versatility as a mathematical object makes it a cross-cutting concept. It is not only a tool for elementary calculation, but has also become a fundamental instrument for describing and solving complex problems.

In relation to the above, Jones (2015) identified two additional conceptualisations beyond the area under the curve: the values of an antiderivative and the concept of accumulation via the limit of Riemann sums (Jones & Ely, 2022; Swidan & Naftaliev, 2019; Wagner, 2018).

These two conceptualisations, in turn, establish a connection between the derivative (rate of change) and the antiderivative (accumulation). In particular, the conceptualisation of accumulation is based on the dynamic relationship of covariation between two variables, in which one variable accumulates as the other varies continuously (Kouropatov & Dreyfus, 2013; 2014). In this sense, the accumulation function can be defined in terms of the antiderivative, since it represents the accumulation of a quantity from its rate of change. This conceptualisation is defined essentially because it simultaneously allows for the Fundamental Theorem of Calculus.

However, this conceptualisation has also been approached since its initial formulation in terms of Riemann sums. Stevens and Jones (2023) conceptualise the definite integral using the Adding Up Pieces (AUP) approach. This interprets accumulation as a sum of elementary quantities associated with small, fine partitions of the domain. This structure consists of three layers: the partition of the interval, the calculation of a target quantity in each subinterval $(f(xi) \cdot \Delta x)$and the total sum of these quantities. When the partitions become infinitesimally small, this sum is represented by a definite integral, thus providing a quantitative meaning to accumulation beyond geometric or analytical interpretations (Caglayan, 2016; Sealey, 2014).

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Similarly, Lehmann (2024), like Borji, Martínez-Planell, and Trigueros (2024), structures the understanding of the definite integral as a cumulative sum of approximate quantities. This is done through contextualised tasks and the implementation of the trapezoidal rule, showing how students develop this understanding progressively from the numerical, graphical, and symbolic levels. In this way, the conceptualisation of accumulation is interpreted not simply as an area, but as the coherent addition of pieces on a partition of the domain, an approach also supported by previous research such as that of McGee and Martínez-Planell (2014).

As a complement to the above, a broader conceptual perspective is the conceptualisation of the accumulation function, understood as the integral function, i.e., as a function defined by an integral with a variable upper limit, expressed as $F(x) = \int_a^x f(t)dt$ (Radmehr & Drake, 2017). This function represents the net accumulation of a quantity whose rate of change is given by f over the interval $[a, x]$. From a didactic perspective, its construction involves the simultaneous coordination of three mathematical objects: the independent variable x, the function that models the accumulated change $f(t)$, and the cumulative function $F(x)$, which requires complex covariational reasoning (Kouropatov & Dreyfus, 2014; Aranda & Callejo, 2017).

Understanding this function from a cognitive point of view does not only involve visualising how the values of f accumulate as x varies, but also interpreting the definite integral as a dynamic process in which quantities generated by the covariation between two magnitudes are incrementally added (Carlson et al., 2010). This understanding requires constructing a mental image in which the variation of x simultaneously determines a new accumulation interval.

However, the integration between the conceptualisation of accumulation and the integral function or accumulation function in turn favours a less superficial internalisation of the Fundamental Theorem of Calculus, by allowing the derivative of the accumulation function to be interpreted as the instantaneous rate of change of the accumulated quantity, provided that the function modelling accumulation is continuous.

In line with this perspective, Polla (2015) describes the conceptualisation of numerical approximation using various methods, undoubtedly based on the principle of refined finite sums. His analysis considers the use of interpolations, as in Simpson's method, or extrapolations, such as the Richardson technique used in Romberg's method, to increase accuracy through successive combinations of approximations. This perspective connects numerical reasoning with algorithmic strategies that model accumulation from a computational approach (Xu et al., 2021), highlighting how efficiency and accuracy depend on the type of integrated function and the method used.

Thus, in this context, the conceptualisation of the average value of a function becomes relevant, as it provides a measure of the function's overall behaviour over a specific interval (Greefrath et al., 2021). This idea, formalised by the expression $\frac{1}{b-a}\int_a^b f(x)dx$, allows the definite integral to be interpreted not only as a process of accumulation of values or numerical approximations, but also as a key resource for determining the representative value of the magnitude described by the function in the interval considered. This conceptualisation emphasises that the average value of a continuous function requires, on the one hand, a geometric interpretation in terms of the area under the curve, implying an advanced understanding of calculus. And on the other hand, a deep understanding of the notion of average in statistical terms (Greefrath et al., 2016).

This link with statistics is established by recognising that this expression generalises the arithmetic mean. In contrast, in statistics, the average of a finite set of data is calculated, and in integral calculus, the average value of a continuous quantity is obtained by evenly distributing the area under the curve over the integration interval. Thus, the average value of a continuous function can be understood as a continuous form of the statistical average, which favours interdisciplinary

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connections with the notion of mathematical expectation in probability, and with the concept of weighted average in physical, economic, or social modelling contexts.

However, understanding the concept of a definite integral is greatly enhanced when it is introduced through real-world situations. Therefore, a complementary conceptualisation is based precisely on this approach. In this sense, Sealey (2014) argues that a "real-world situation" involves authentic contexts in which continuously varying quantities are involved, such as speed, force or pressure, and whose modelling requires accumulation processes. Along the same lines, Nguyen and Rebello (2011) reinforce this conceptualisation, also basing their argument on contextualised physical problems, such as the sphere problem, in which angular displacement must be considered to calculate work, or the arc problem, which requires an interpretation of function graphs such as $cos(\theta)$to establish the electric field.

This type of situation challenges students to identify the variables involved and to break down, interpret, and reconstruct the conceptual framework of integral calculus. Through the analysis of real phenomena, in which multiplicative schemes interact throughout a given domain. In this way, the "area" is not only seen in a geometric sense but is extended to an algebraic understanding.

A similar approach can be seen in the study by Busrah and Pathuddin (2021), who articulate the learning of the volume of solids of revolution through an ethnomathematical lens. This research distinguishes itself by treating solids not as abstract figures but as three-dimensional objects that emerge from the formal analysis of traditional foods from the Buginese and Makassarese cultures. Using polynomial functions obtained by interpolation. The authors mathematically model these foods, such as lammang, paso, cucuru, and pisang ijo, and employ classic methods of integral calculus, including discs, washers, and cylindrical shells, to estimate their volumes (Cetin & Dev, 2023).

This type of research allows us to introduce a last conceptualisation of the definite integral, linked to the formation of solids of revolution (Paul et al., 2024; Mofolo-Mbokane et al., 2013). This conceptualisation transcends and reconfigures the area under the curve as a process of three-dimensional accumulation of differential volumes, generated by rotating a two-dimensional region around an axis. For his part, Knudson (2018) offers an alternative view by exploring solids of revolution cut by planes parallel to the axis of rotation, highlighting how different ways of sectioning the solid reveal unexpected analytical and visual properties, as in the case of Gabriel's horn, which has finite volume but infinite surface area.

Finally, Ďuriš (2020) uses computational tools (such as MATLAB and the Monte Carlo method) to visualise and integrate surfaces associated with defined functions, showing how the integral can be used not only in geometric terms but also to solve measurement problems in multidimensional spaces.

In this regard, it is essential to clarify what is meant by conceptualisation in the context of this research. We understand this term as the process of constructing, organising, and evolving meanings around a mathematical concept (Godino et al., 2007). This process involves acquiring formal definitions, using semiotic registers, and appropriating meanings that may be mediated by context. These meanings give rise to forms of thought that can range from the processual to the structural, conceiving mathematical concepts as action processes as well as abstract entities that can be mentally manipulated (Tall, 2013; Sfard, 1991).

Therefore, the results and their analysis allowed us to summarise the conceptualisations presented in Table 2. These show how, over time, various conceptions, meanings and mental schemas have been consolidated and formalised around the concept of the definite integral. These conceptualisations reflect the diversity of ways in which this mathematical object has been

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interpreted by the mathematical community, accounting for its semantic complexity and the different approaches from which it has been addressed in educational and research contexts.

Table 2. Conceptualisations of the definite integral found in the literature

Category	The definite integral as....	Authors	Code
Geometric	The area of a region bounded by the graph of a continuous non-negative function and the x-axis in a specific interval corresponding to the limits of integration.	(Kontorovich, 2023; Kontorovich & Li, 2022)	GM
Antiderivative	The process of reconstructing a function F whose derivative is f. That is, such that $F'(x) = f(x)$ for all x in the domain considered.	(Greefrath et al., 2021; Jukić & Dahl, 2014)	AN
Average value of the function	The measure of the average values of a function over a specific interval (expressed mathematically $\frac{1}{b-a}\int_a^b f(x)dx$).	(Greefrath et al., 2021)	VP
Accumulation	The total sum of infinitesimal quantities over an interval. This sum can be approximated using Riemann sums $\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$ or darboux sums (supremes or infima).	(Oehrtman & Simmons, 2023; Soto & Oehrtman, 2022; Park et al., 2013)	AC
Approximation	A numerical approximation process using the trapezoidal method or Simpson's method.	(Brandt, 2016; Polla, 2015)	AP
Accumulation function	A relationship that allows us to observe how two quantities change simultaneously: the independent variable (the upper limit of the integral) and the cumulative value of the function. Formally, the accumulation function is defined as: $F_a(x) = \int_a^x f(t)dt$	(Kouropatov & Dreyfus, 2014; Palha & Spandaw, 2019)	FC
Real-world situation	Physical situation (Hooke's law, for example: the energy needed to stretch a spring). Functional situation (speed *vs.* Time; speed *vs.* Distance). For example, estimating the distance a gorilla fell in half a second. Biological situation (calculating the flooded area). For example, the overflowing of the Imboassica Lagoon.	(Sealey, 2014; Alves et al., 2019)	SR

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Solids	The volume generated by rotating a region of the plane around an axis (usually the x-axis or y-axis). This volume is calculated by integrating the area of the cross sections of the solid along the rotation interval. One of the formulas for expressing this volume is $V = \pi \int_a^b [f(x)]^2 \, dx$	(Gulkilik, 2022; de Carvalho et al., 2024)	SO

Following the description of each of the previously identified conceptualisations, the applications chosen for the tasks in each of these categories were also considered in general terms, as well as the semiotic representations (see Table 3). In addition, some of the connections established with other mathematical concepts were also considered (see Table 4).

Table 3. Contexts of application of tasks and types of semiotic registers used in conceptualisations of the definite integral (2010-2024)

Code	Context of task application	Semiotic representation records
GM	Biological Physical Mathematical	Graph Geometric Numerical Algebraic Tabular
AN	Biological Mathematical	Graph Numerical Algebraic
VP	Physical Mathematical	Graph Numerical Algebraic
AC	Biological Physical Mathematical Field of economics	Graph Geometric Numerical Algebraic Tabular Written (problem situations)
AP	Physical Mathematical	Graph Geometric Numerical Algebraic Tabular

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FC	Physical Mathematical	Graph Geometric Numerical Algebraic Tabular Written (problem situations)
SR	Biological Physical	Numerical Algebraic Written (problem situations)
SO	Biological Physical Cultural	Graph Geometric Numerical Algebraic Written (problem situations) Iconic

The information presented in Table 3 is complemented by a graphical summary using a bar chart, which highlights the identified semiotic records (see Figure 2).

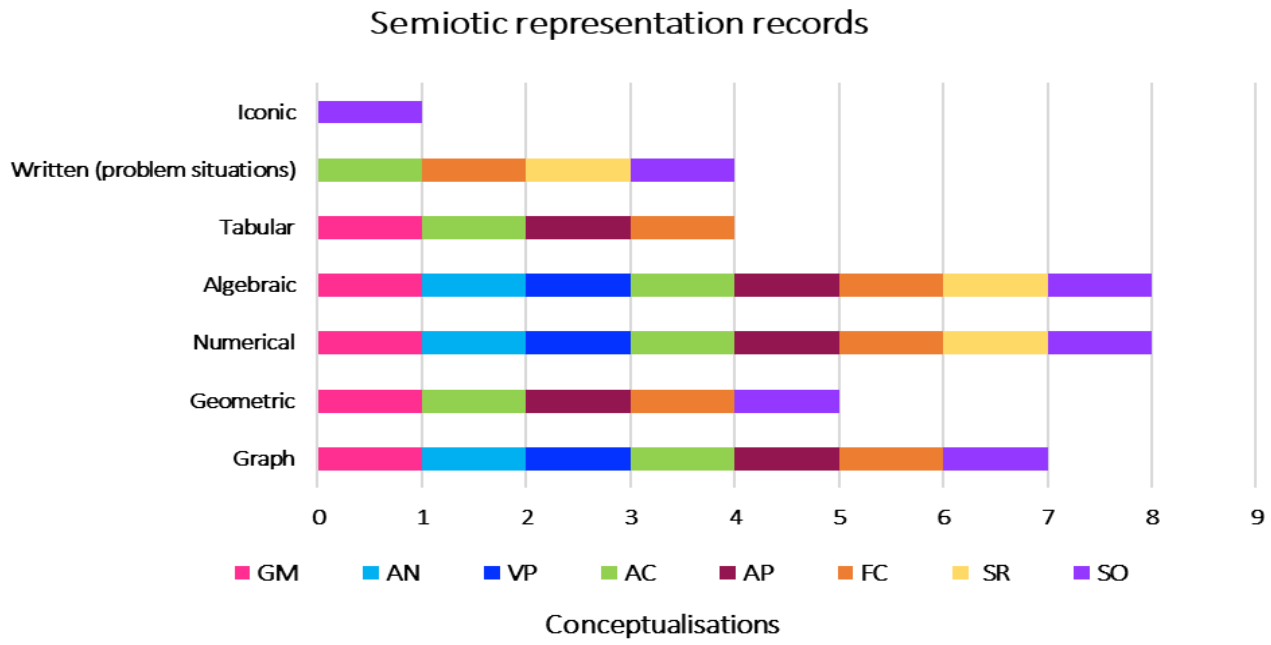

Figure 2. Semiotic records present in conceptualisations.

In addition to the variety of semiotic registers, it is also pertinent to examine the contexts in which mathematical tasks are framed. These contexts provide information about how the definite integral is introduced and applied, revealing whether the tasks are situated in purely mathematical settings or extend to interdisciplinary and real-world applications. Figure 3 illustrates the distribution of the application contexts of the mathematical tasks analysed, complementing the information provided in Table 3. Although it is not the central purpose of our research to identify which of these contexts predominates, the analysis carried out can provide future research with a useful basis for

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further investigation into how the nature of the context influences the teaching and learning of the definite integral.

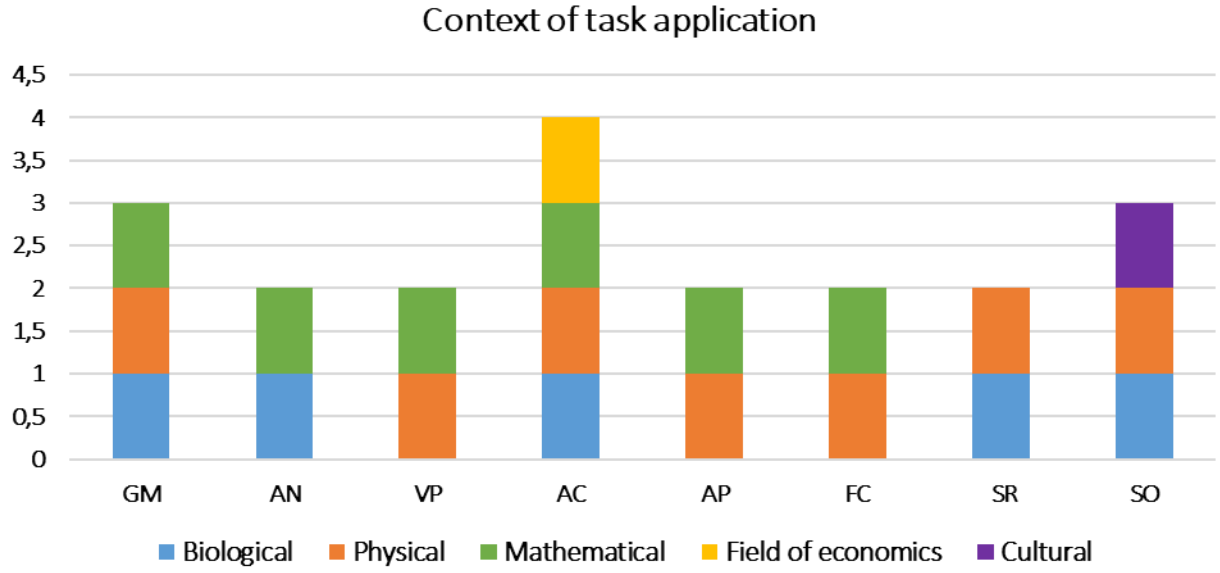
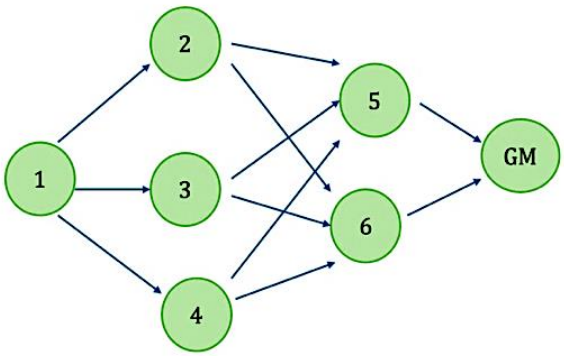


Figure 3. Contexts for applying mathematical tasks

Within the selected studies, relationships and connections between the conceptualisations of the definite integral were evident. From these interactions, a set of mathematical concepts emerged, forming a structural network of interrelated elements, fundamental to both understanding and teaching this mathematical object. Among the concepts identified are: 1. Area of a geometric figure; 2. Continuous function; 3. Interval (set of integers) ; 4. Partition of an interval; 5. Riemann sums; 6. Limit; 7. Slope; 8. Rate of change; 9. Variation between quantities; 10. Rate of change; 11. Average rate of change; 12. Derivative; 13. Antiderivative; 14. Fundamental Theorem of Calculus; 15. Magnitude; 16. Sums; 17.Compact Sums; 18.Darboux Sums; 19. Discrete Quantity; 20. Convergence; 21. Infinitesimal; 22. Continuous Quantity; 23. Increment (Δx,Δy) 24. Area of a trapezoid; 25. Parabola; 26. Numerical estimation; 27. Simpson's method; 28. Romberg's method; 29. Length of a curve; 30. Area under the curve; 31. Volume; 32. Upper limit; 33. Lower limit; 34. Hooke's law; 35. Velocity; 36. Distance; 37. Density; 38. Product of variables; 39. Limited region; 40. Solids of revolution; 41. Disk method; 42. Shell method; 43. Cylinder method and 44. Differential notation.

Table 4. Connection network with other mathematical concept

Code	Related mathematical concepts
GM	

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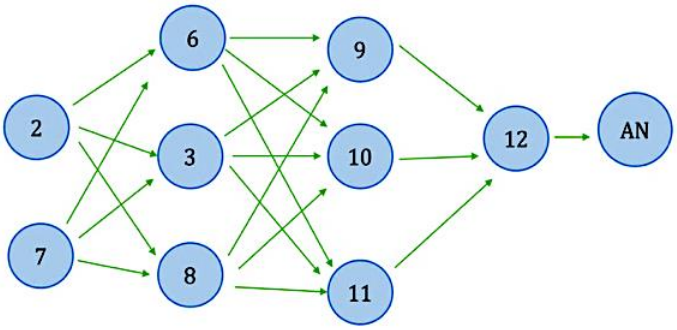
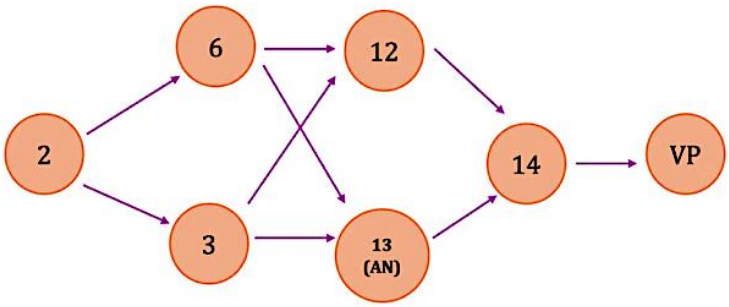
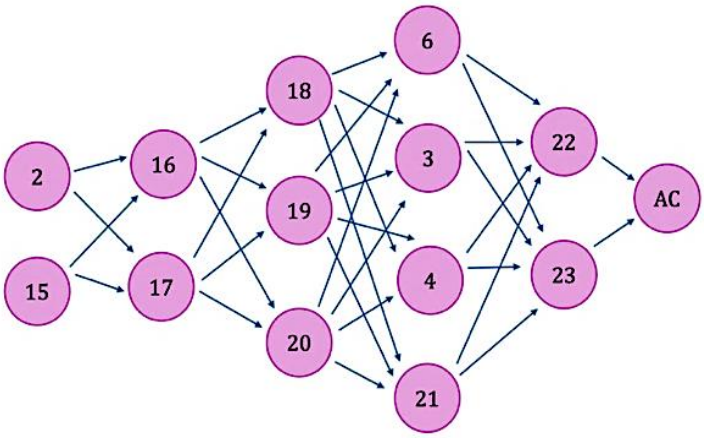
AN



VP

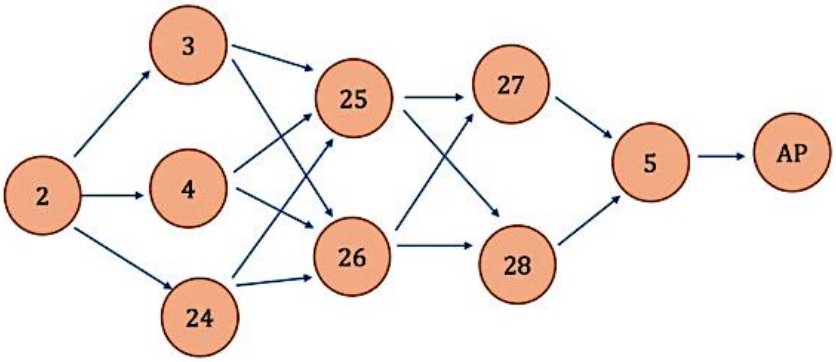


AC



Note: In this conceptualisation, the presence of Darboux sums (18.) gives rise to Riemann sums (5.), thus establishing a connection with the conceptualisation GM

AP



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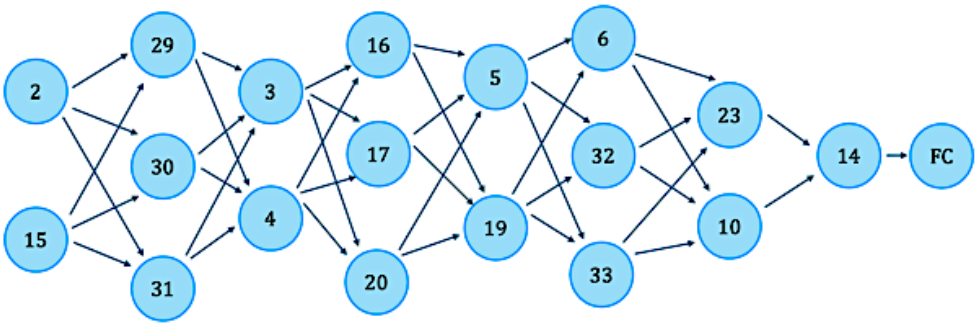
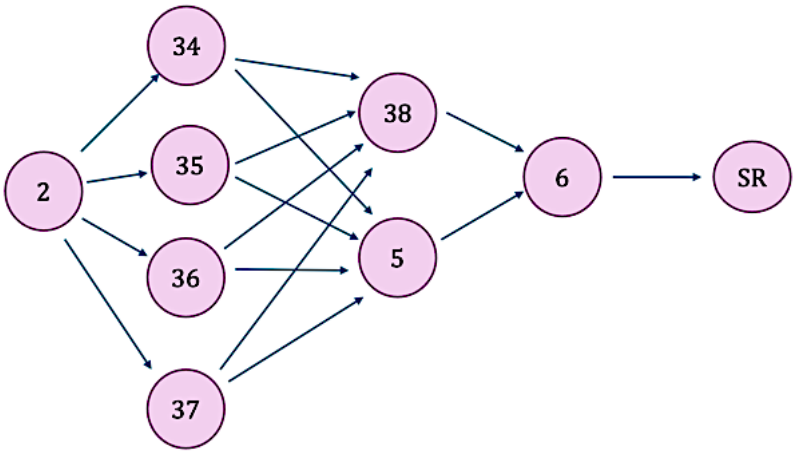
Note: This conceptualisation is also connected to GM

FC



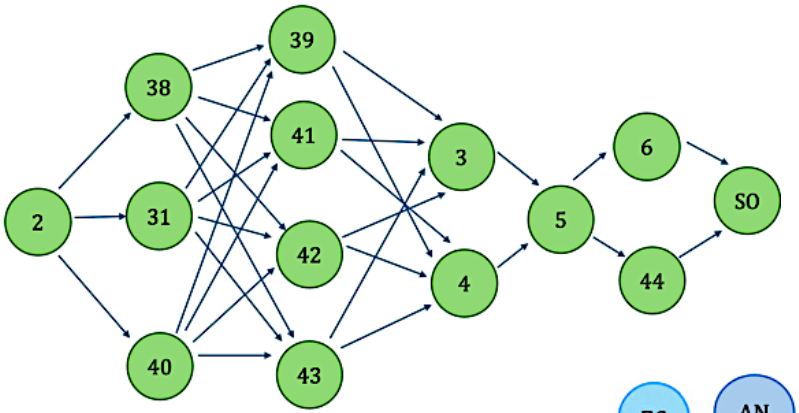
Note: This conceptualisation is also connected to GM

SR



Note: This conceptualisation is also connected to GM ; AN y AC

SO



Note: This conceptualisation is also connected to FC ; AN y AC

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After constructing a network of connections among other mathematical objects, the VOS viewer software was used to generate a co-occurrence network of terms from articles indexed in databases such as Scopus. The objective of this stage was to identify the predominant conceptualisations surrounding the concept of the definite integral. This network allowed us to visualise, in a structured way, the relationships among key terms such as the definite integral, the Fundamental Theorem of Calculus, and Riemann sums, among others.

The network analysis revealed thematic clusters that reflect different approaches to the concept: from its theoretical and formal interpretation to its link with student learning and applications (see Figure 4). This visualisation not only facilitated the identification of recurring lines of research, but also allowed gaps in the literature to be detected, such as the lack of articulation between educational approaches and the theoretical foundations of integral calculus.

Consequently, the bibliometric network constructed served as a relevant input for defining the analytical categories of this study and substantiating the need to further explore the various mathematical conceptualisations of the definite integral from an educational perspective.

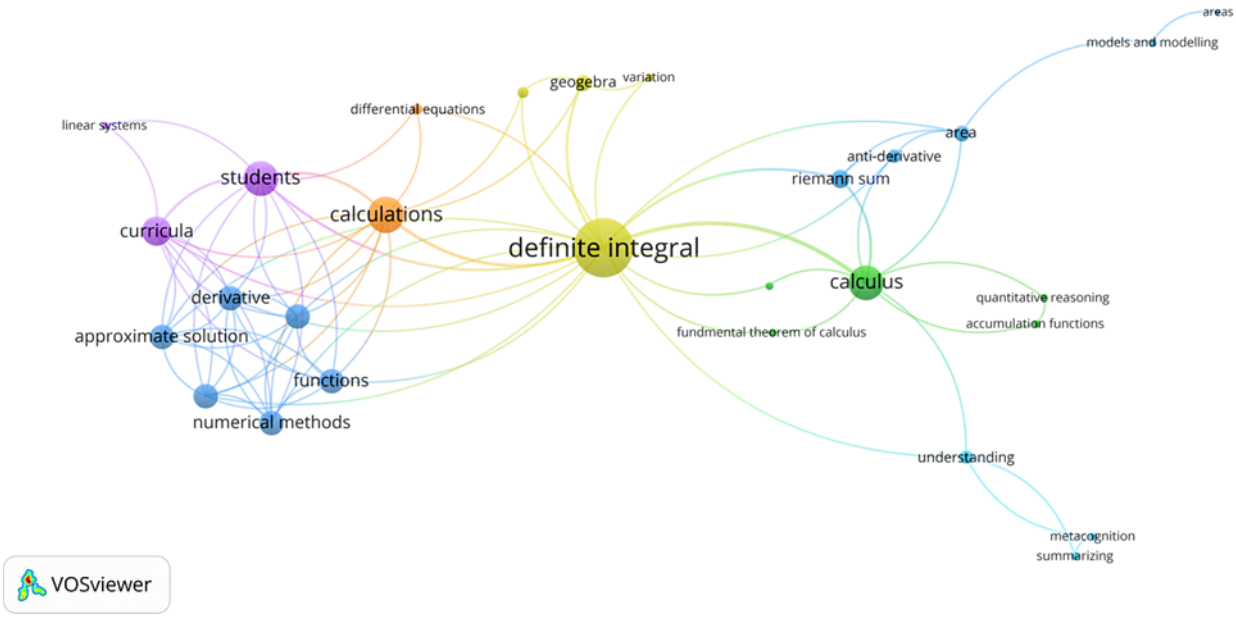

Figure 4. Keyword co-occurrence network generated with VOSviewer software.

This is how the network structure shows variations in the degrees of co-occurrence between keywords, reflecting their centrality and weight in the analysed literature. The node “integral definite” stands out for its high frequency and density, acting as a connecting hub between the different clusters. Its position indicates the relevance of the mathematical object as a point of intersection between lines of research oriented towards mathematical formalisation and those focused on student learning.

In this last line, we see the term GeoGebra, linked to concepts such as understanding. This suggests that there is a field of study in the literature focused on the use of digital tools to enhance learning of the concept of the definite integral. This is particularly true in relation to its interactive graphical visualisation and approximation using Riemann sums.

On the other hand, a node corresponding to curricula is identified and connected to the term students. Although terms such as mathematics education, teaching or tasks do not appear explicitly, the presence of these links suggests an orientation towards curriculum design. However, this node maintains a weak connection to terms specific to the formal approach to calculus, such as limit or

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Riemann sum, indicating a lack of articulation between the conceptual structure of the mathematical object and its curricular implementation.

Overall, the most representative nodes reveal the coexistence of several of the conceptualisations addressed in this manuscript. However, the network shows a fragmented distribution of these conceptualisations, with formal, technological and curricular approaches forming thematic communities that are poorly connected to one another. This segmentation suggests that, although there is great richness surrounding the mathematical object, there is still weak articulation between the different dimensions from which its teaching and understanding are approached.

4.2. Digital tools implemented in the conceptualisations of the definite integral

Now, complementing the findings of the bibliometric analysis network generated by VOSviewer regarding the technological component. Of the 64 articles analysed in the systematic review, only 36 incorporate technological tools across the eight conceptualisations described above (see Figure 5). These results show a preference for easily accessible, visually appealing resources, reflecting the importance of dynamic graphic representation environments.

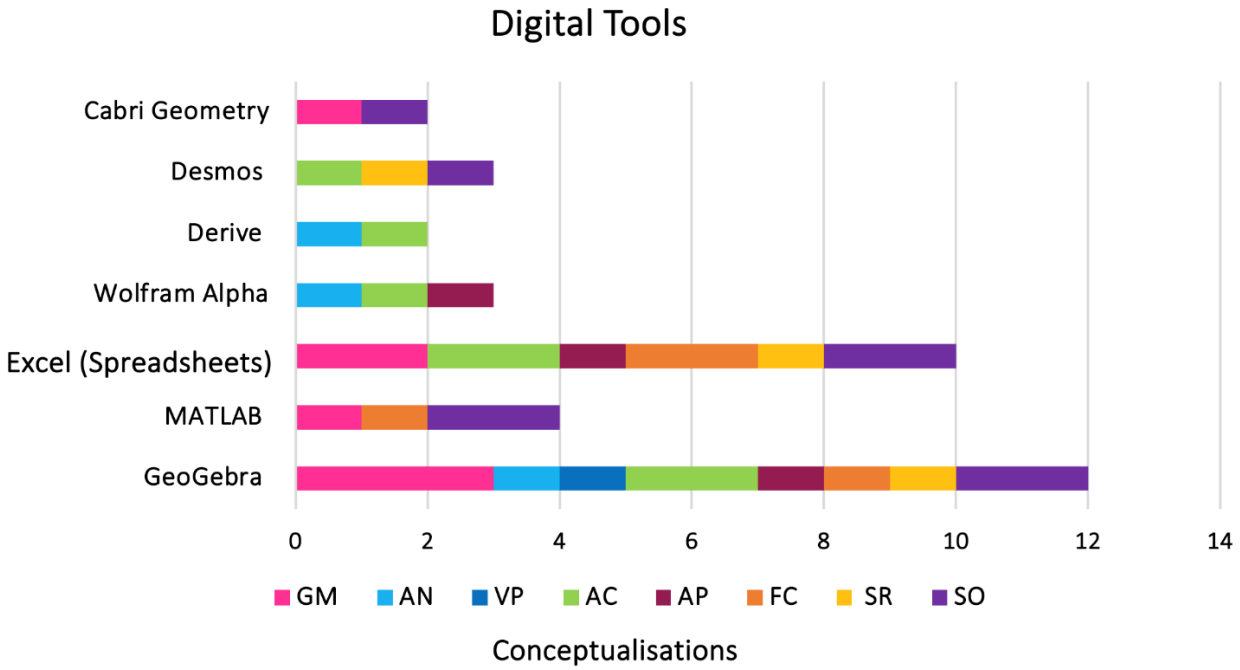

Figure 5. Digital tools used in the researches on conceptualisations of the definite integral (2010–2024)

To better understand the data, it is also presented in percentage terms (see Figure 6), allowing a clearer assessment of the proportion of use of each tool across the studies reviewed. The results of the review show that the most widely used digital tool in research on the definite integral during 2010–2024 was GeoGebra, with a 33% share, followed by Excel spreadsheets with 28%.

This predominance is largely due to the extensive functionality offered by GeoGebra, whose dynamic features allow users to graph functions, shade areas under the curve, and, through animations, dynamically identify partitions of a function in a given interval. It also enables the creation of interactive applets that allow parameters to be manipulated and tasks to be designed in real-life contexts. These features not only facilitate the visualisation of the definite integral but also allow it to be approximated numerically by calculating sums or averages of discrete values representing the function, thereby promoting understanding of the accumulation process.

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Complementarily, Excel plays a role focused on numerical calculation, as it allows data to be organised into tables and performs operations such as sums and averages of discrete values, making it possible to approximate the area under the curve and estimate the definite integral. In addition, it also encourages students to systematically explore accumulation processes, reinforcing the connection between algebraic and numerical approaches.

Other digital tools also played a role in the studies reviewed, albeit to a lesser extent. MATLAB (11%) stood out thanks to its ability to perform high-precision symbolic and numerical calculations. Its libraries allow definite integrals to be approximated using different quadrature methods, as well as solving modelling problems in engineering contexts, highlighting its value in dealing with complex situations.

Wolfram Alpha (8%) was used as a resource for verification and conceptual exploration, offering immediate symbolic and graphical responses that facilitate validating manual procedures and understanding the relationship between antiderivatives and areas under the curve. Finally, Derive (6%), although no longer in use, appeared in some older studies, providing a computational algebraic environment that facilitated the transition from symbolic manipulation to graphical representation.

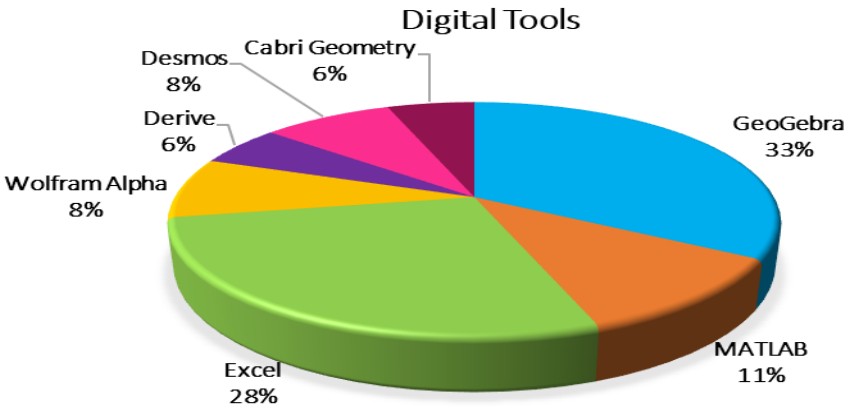

Figure 6. Percentage of digital tools used in the studies reviewed

This situation is striking because, despite the fact that we are immersed in an era characterised by technological advances and the widespread availability of digital resources, the integration of these tools into the research and teaching of the definite integral remains limited. This situation highlights a gap between the pedagogical potential of digital technologies and their effective application in education.

5. Discussion

The findings of this study provide an understanding of the semantic complexity of the concept of the definite integral, understood as the multiplicity of meanings attributed to it across different educational and disciplinary contexts. This diversity shows that its teaching cannot be reduced to a single interpretation, since identifying diverse conceptualisations that coexist and articulate with one another provides a broader and more robust framework for its understanding. Incorporating this conceptual variety is cross-cutting, as it generates greater richness in the mental schemas that students and teachers can construct cognitively (Tarr & Maharaj 2021; Brijlall & Ndlazi, 2019; Martínez-Planell & Trigueros, 2020).

Among the conceptualisations identified in the literature, eight forms stand out. It should be noted that, although many of these conceptualisations incorporate Riemann sums as a key tool for estimating or constructing the value of the integral, there is a notable absence of a formal definition of these sums from an epistemological perspective. This absence reinforces the findings of Nilsen and Knutsen (2023), who showed that a high percentage of students conceive of definite integrals

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primarily as areas or antiderivatives, rather than as cumulative sums. This creates difficulties when linking Riemann sums to antiderivatives.

In particular, the basis for the concepts of actual infinity (as a completed whole) and potential infinity (as a developing process) is not clearly addressed (Roa & Oktaç, 2014). This conceptual gap has been highlighted by authors such as Borji et al. (2024), who warn of the difficulties students face when attempting to understand the transition from a finite to an infinite sum without clear theoretical mediation. This omission weakens the construction of consolidated meanings with a structure that has the potential to establish deep connections.

Similarly, Díaz-Chang and Arredondo (2022) and Ely (2021) state that the concept of infinity is highly counterintuitive and requires detailed treatment in the teaching of calculus. The absence of a precise definition of the type of infinity underlying Riemann sums can lead to ambiguity. For example, in understanding the limit process (Nilsen & Knutsen, 2023; Hong, 2023a). This brings with it the difficulty of internalising convergence towards a defined value.

This conceptual difficulty is not an isolated phenomenon; it is also evident in the results of the co-occurrence analysis conducted with VOSviewer. This analysis reveals a partially disconnected distribution between components conceptual and pedagogical. In particular, the co-occurrence analysis shows that some conceptualisations of the definite integral emerge, such as the geometric, the antiderivative, the accumulation function and accumulation from Riemann sums. These conceptualisations do not form an integrated network, but are grouped into distinct clusters, i.e., without a direct connection. This suggests a possible correspondence with the theoretical difficulties that students face when trying to understand infinity and limit processes.

This interpretation is supported by the findings of Greefrath et al. (2021); Mateus-Nieves and Font Moll (2021), who identify multiple meanings associated with the definite integral and warn of the difficulty students have in moving between them. Such a disconnect between the different meanings not only limits conceptual articulation but also affects the conceptual construction and formal definition, hindering its abstract understanding (Grundmeier et al., 2006).

Furthermore, this partially disconnected distribution also permeates technological and curricular resources. In this regard, the distribution of digital tools used in the studies reviewed (see Figure 6) shows that, although digital tools such as GeoGebra (33%) and Excel (28%) are used, which stand out for their ability to visualise, explore and dynamically approximate concepts such as definite integral. Their implementation does not guarantee an integrated understanding of the concept, a premise supported by authors such as Tatar and Zengin (2016). However, its use facilitates the transition between graphical, numerical and algebraic representations, thus promoting noesis and generating conversions between records.

This situation is illustrated in Figure 2, which shows the distribution of the different semiotic registers that emerge in the studies analysed. As can be seen, there is a prevalence in the use of the algebraic and numerical system, which coincides with what Garcia-Garcia and colleagues (2025) point out when they explain that in the teaching and learning of definite integrals, these registers tend to be privileged, while graphs and tables receive considerably less attention. This invites us to reflect, in our educational and research practices, on how certain registers are favoured, generating bias and detriment to systems that favour analytical generalisation, considered one of the most complex cognitive compositions for students (Martínez-Planell & Trigueros, 2020).

In addition to this, the results obtained in the conceptual analysis show that, of the eight conceptualisations identified at university level, the geometric, antiderivative and approximation conceptualisations predominate. This finding aligns with that reported by Jones (2015) and Sealey (2014), who note that these perspectives have traditionally been privileged in the teaching of

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calculus. However, over the last four years, the conceptualisations of solids of revolution, accumulation, and accumulation function have strengthened. This trend has also been identified in recent studies (Borji & Martínez-Planell, 2023; Hayes, 2024). From a pedagogical standpoint, this emerging change calls for rethinking the teaching of definite integrals, promoting a transition from traditional approaches to proposals that integrate these perspectives in a balanced manner.

In line with this, it is essential to highlight the importance of the phenomenological context in task design, as pointed out by various authors (Busrah & Pathuddin, 2021; Sealey, 2014) and as shown in Figure 3, because although the literature repeatedly acknowledges this problem, it is still not being addressed with the rigour necessary to authentically link mathematical meanings with real-life situations, not restricted solely to physical or biological situations, but also to social, economic or technological issues. In line with this, the integration of technology takes on a transcendental role, since through a didactic design that articulates dynamic visualisation, numerical calculation and modelling in contexts, so that each tool contributes to shaping the different conceptualisations of the definite integral, it will be possible to strengthen its encapsulation and, consequently, promote apprehension conceptual and semiotic.

6. Conclusion

Therefore, the results obtained indicate that the curricular component also plays a decisive role, as evidenced by the classroom research analysed in this study. It is observed that higher education curricula, in relation to the mathematical elements that must be addressed in class, do not always relate sufficiently to the eight conceptualisations. This reduces students' opportunities to explore the meanings of the definite integral beyond the area under the curve or integration procedures. This provides an insight into why students tend to master formulas and algorithms but fail to fully consolidate their understanding of the meanings of this mathematical object.

Hence the need to propose modifications and redesigns to the curriculum so that tasks reinforce the connection between graphical and symbolic representations, as well as the distinction between derivability and differentiability. Similarly, the epistemological dimension of infinity should be intentionally reincorporated, including the assimilation of limits as a process and the distinction between potential and actual infinities. In this way, students can transcend the repetitive application of procedures and construct conceptual scaffolding that merges the notion of limits and Riemann sums.

Accordingly, it is essential to innovate in the teaching strategies that guide the teaching process, orienting them towards fostering collective discussion. For example, through the implementation of theoretical frameworks such as APOS or Hypothetical Learning Trajectories, collaborative problem solving, the conscious use of technology, and metacognitive reflection. All of this is done to broaden students' learning processes and foster a critical, motivating environment. In this environment, the classroom becomes a space of trust, and the latent skills of each of the actors involved in the mobilisation of knowledge are enhanced.

7. Limitations and Challenges of the Research

Although the knowledge generated by this study is significant for mathematics education, several limitations in the methodological development and data extraction must be acknowledged. First, the review was restricted to articles published in English, which may have excluded valuable research in other languages, particularly in Ibero-American contexts where significant contributions to the field are made. Secondly, Master's and Doctoral theses, as well as conference proceedings and presentations, were excluded. Although these do not always undergo rigorous peer review, they often contain novel findings and emerging teaching proposals. Finally, the possible restrictive effects of the PRISMA criteria, such as the focus on documents indexed in refereed journals, leaving out

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studies that, although less visible in high-impact circuits, may include relevant contributions to understanding the conceptualisations of the definite integral and the use of technology in its teaching and learning.

Given these limitations, several challenges emerge for future research: broadening the spectrum of sources and languages considered to enrich the cultural and epistemological diversity of the findings; integrating grey literature and non-indexed works that provide innovative perspectives; and designing methodologies that overcome the rigidity of review protocols, so that it is possible to capture the complexity and richness of teaching and learning practices in integral calculus in different educational contexts.

8. Suggestion

This systematic review of the literature highlights the relevance of different conceptualisations of the definite integral for transforming the cognitive construction of this mathematical object. Therefore, based on the findings, it is suggested that further research be conducted into the design of educational strategies that coherently articulate the different epistemic, semiotic, technological, and didactic perspectives associated with the definite integral. In particular, it is recommended to develop mathematical tasks that integrate these components in order to achieve more refined levels of abstraction. These proposals could significantly transform students' prior conceptions, facilitating a more solid construction grounded in real contexts. Additionally, it is suggested that future revisions expand their scope to include relevant regional studies, doctoral theses, and innovative approaches documented in languages other than English. This expansion would make visible theoretical and practical perspectives that, due to their contextual characteristics, are often excluded from international academic databases.

Declarations

Author Contributions. L.P.M.B.: Literature review, conceptualization, methodology, data analysis, review-editing and writing, original manuscript preparation. G.M.S.: Literature review , methodology, data analysis. C.D.F.: Methodology, data analysis. All authors have read and approved the published on the final version of the article.

Conflicts of Interest. The authors declare no conflict of interest.

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Ethical Approval. The authors declare that this study is a systematic review of literature based exclusively on previously published research. Since no original data were collected and no direct interaction with human participants occurred, ethics committee approval was not required.

Data Availability Statement. The data supporting the results of this study were generated during the research process and are available upon reasonable request to the author. Derived data supporting the results of this study may also be provided upon request.

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