

Research Article

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Deep Learning-Oriented Mathematics Learning: Strengthening Critical Thinking in Junior High School

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Abstract

Background/purpose. Deep learning-oriented mathematics learning to strengthen students' critical thinking is an urgent need. Strengthening critical thinking prepares students to face the challenges of an era of globalisation marked by complexity. This article has two research objectives. 1) To describe the process of deep learning-oriented mathematics learning. 2) To explore strategies for strengthening students' critical thinking in deep learning-oriented mathematics learning.

Materials/methods. This article is a qualitative-ethnographic study. The research was conducted at SMP Negeri 3 Kartasura, Sukoharjo, Central Java. The research period was from February 2025 to July 2025. The research data were primary and secondary. The research subjects were the principal, mathematics teachers, and students of class VIII B in the first semester at the research location. Data collection was conducted through interviews, observations, and document analysis. The researcher was the key instrument. Data validation was conducted through source and method triangulation. Data analysis was conducted through induction.

Results. 1) The deep learning-oriented mathematics learning process to strengthen critical thinking in junior high school students, namely a) preliminary activities; b) core activities; and c) closing activities. 2) Strategies to strengthen students' critical thinking in deep learning-oriented mathematics learning in junior high school, namely strategies to strengthen the indicators of a) analysis, b) evaluation, and c) inference.

Conclusion. Deep learning-oriented mathematics learning in junior high schools is designed to strengthen students' critical thinking—analysis, evaluation, and inference—through structured objectives, components, syntax, and targeted strategies.



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1. Introduction

Traditional mathematics learning that emphasises procedural memorisation has proven inadequate in preparing junior high school students to tackle complex tasks and contextual problems that require deep thinking. Therefore, a deep learning orientation that places problem solving, metacognition, and contextual projects at the core of learning is essential to strengthen students' critical thinking skills (Laakso et al., 2021; Grover et al., 2020). Recent studies reaffirm that mathematics education must shift from procedural teaching to deeper conceptual engagement to improve learners' analytical and reflective thinking abilities (Nguyen & Lee, 2023; Praveen & Malik, 2022). In addition, empirical evidence from previous studies shows that learning strategies that emphasise productive struggle, collaborative reasoning, and metacognitive reflection significantly enhance students' achievement and higher-order thinking skills (Lye & Koh, 2022; Wang & Kokotsaki, 2021). Therefore, the integration of deep learning into mathematics education has become a strategic priority for strengthening the quality of national mathematics education.

Strengthening critical thinking among junior high school students is urgent to prepare them for the complexities of globalisation. Critical thinking enables students to analyse problems in depth, evaluate possible solutions, and make logical decisions based on relevant data (Adams & Hewitt, 2022; Lai, 2021). At the junior high school level, critical thinking skills are fundamental to academic success and character development, particularly in navigating learning challenges that demand innovative and adaptive thinking (Burke et al., 2021; Koh et al., 2019). Enhancing students' critical thinking skills is also essential for developing a deeper understanding of mathematics concepts and applying knowledge in real-life problem-solving.

However, preliminary observations and interviews conducted at the research site indicate that the implementation of deep learning in mathematics remains limited and faces several challenges. Although deep learning has the potential to stimulate students' critical thinking, its application is often superficial, limited to basic concepts and lacking deeper cognitive engagement. Furthermore, mathematics teaching continues to rely heavily on conventional teacher-centred instruction, limiting opportunities for students to engage in independent exploration, collaborative reasoning, and reflective thinking. Interviews with the headmaster and a mathematics teacher further reveal that more than 70% of students struggle with thinking critically, while 72% remain passive during classroom learning. These conditions align with previous studies showing that teacher-centred approaches hinder active participation, reduce engagement, and weaken the development of students' critical thinking skills (Adams & Hewitt, 2022; Wang & Kokotsaki, 2021).

A major contributing factor is the limited competence of mathematics teachers in implementing deep learning. At least 55% of mathematics teachers at the site have not received adequate training related to deep learning principles and practices. As a result, classroom instruction remains heavily procedural, lacking the integration of exploration-oriented activities essential for developing critical thinking. This challenge is compounded by insufficient technological facilities and learning resources needed to support deep learning-based instruction. Similar gaps in teacher readiness and instructional design have been highlighted in recent international studies, emphasising that successful implementation of deep learning requires systematic professional development and supportive school ecosystems (Lamon et al., 2022; Pettersson & Olofsson, 2023).

International studies show promising outcomes regarding the integration of deep learning into mathematics education. Research conducted in the United States, Australia, and Finland demonstrates that deep learning approaches effectively improve students' problem-solving and critical thinking abilities by deepening their understanding of abstract mathematical concepts (Zheng et al., 2020). Similarly, studies in Singapore and Canada show that deep learning, when combined with collaborative projects and technology-enhanced tasks, strengthens students' analytical skills and

supports meaningful engagement with real-world mathematical problems (Cheng et al., 2021; Leijen et al., 2021). More recent global findings also highlight that deep learning-oriented mathematics instruction fosters long-term conceptual mastery and higher-order reasoning when supported by a responsive pedagogical framework (Huang & Shimizu, 2023; Romero-Ariza, 2022).

Although previous studies have demonstrated the potential of deep learning in mathematics education, significant gaps remain in how deep learning can be optimally integrated to enhance critical thinking, particularly at the junior high school level—particularly in contexts where teacher readiness, instructional design, and technological support remain limited. Few studies have provided a comprehensive description of the learning process that incorporates deep learning principles and explores strategies that directly strengthen students' critical thinking in mathematics classrooms. This gap underscores the need for research that examines both pedagogical implementation and its cognitive impact on learners.

Based on this gap, the present study offers an alternative solution through deep learning-oriented mathematics learning to strengthen junior high school students' critical thinking. Deep learning facilitates the analysis of complex learning indicators, enables pattern detection, and supports real-time feedback to improve learning quality. Critical thinking, as an essential intellectual process, encompasses the ability to analyse, evaluate, interpret, and synthesise information to formulate sound judgments.

Therefore, this study has two objectives. First, to provide a comprehensive description of the deep learning-oriented mathematics learning process implemented in junior high schools. Second, to explore strategies that effectively enhance students' critical thinking skills within the context of deep learning-oriented mathematics education. These objectives aim to address existing gaps in the literature and contribute practical insights for the development of deep learning-based mathematics instruction.

2. Literature Review

2.1. Characteristics of Junior High School Students

Junior high school students are in a transitional stage of development characterised by intensive physical, cognitive, social, and emotional changes. Physically, students experience accelerated growth and early puberty, which can cause feelings of awkwardness and fatigue (Song & Hong, 2026; Ahmetoglu et al., 2020). Cognitively, students begin to enter the formal operations stage, capable of abstract thinking, considering various perspectives, and performing hypothetical and deductive reasoning (Attanasio et al., 2020; Barber & Olsen, 1997). The psychosocial stage focuses on identity search and role exploration, which allows students to develop self-awareness and value choices independently (Zhu et al., 2025). Other socio-emotional aspects involve a strong need for peer acceptance, sensitivity to criticism, mood swings, and adolescent egocentrism, such as the 'imaginary audience' and 'personal fable' (Zubtsova et al., 2024).

Junior high school students have been shown to possess adaptive advantages that can be leveraged in mathematics learning. The development of abstract thinking skills has been shown to facilitate critical thinking, the resolution of complex problems, and logical argumentation (Liu et al., 2025; Hoyt et al., 2019). Students have been shown to possess a high level of curiosity, creativity, and enthusiasm for authentic learning experiences (Shin et al., 2025; Weisleder et al., 2019). In an academic environment that fosters positive self-identity, validates personal experiences, and facilitates collaborative learning, students can optimise their personal strengths, enhance their self-confidence, social adaptability, and academic engagement (Ran et al., 2025). Interventions aimed at enhancing the capacity of educational institutions to promote students' psychological well-being have been shown to improve the psychosocial functioning of junior high school students (Schürer et

al., 2025). Psychosocial functioning is a significant indicator of students' ability to achieve optimal performance and overall well-being (Zhang et al., 2024).

2.2. The Essence of Mathematics Learning

The essence of mathematics learning encompasses not only the product aspect, but also active thinking processes and scientific attitudes, including curiosity, logic, and cooperation that support the development of students' mathematical abilities (Ion & Popescu, 2024; Hoffman & Even, 2023). Operationally, a distinction is made between direct objects (facts, concepts, principles) and indirect objects (attitudes towards mathematics, abstract thinking skills, accuracy, and problem-solving) (Weingarden & Heyd-Metzuyanım, 2024; Bass, 2017). Comprehensive learning indicators include the ability to restate concepts, provide examples and non-examples, present mathematical representations, determine necessary and sufficient conditions, and apply concepts in real-world problem solving (Mityushev et al., 2024; Balaguer, 2009).

Problem-solving, creativity, and communication-oriented mathematics learning not only helps students understand concepts in depth but also encourages the development of critical and reflective thinking skills. For example, exploring patterns and relationships, independent discovery, and the use of intuition strengthen students' ability to construct evidence, generalise, and solve problems in their own way, and to foster logical, systematic, and flexible thinking (Liquet et al., 2024; Rodi, 2024). The application of mathematics learning, in an active learning environment, encourages curiosity, supports reflection, and allows students to express ideas symbolically and verbally. All of this strengthens students' mathematical communication skills and conceptual understanding (Boronenko & Fedotova, 2023; Petruk et al., 2023).

2.3. Integration of Deep Learning in Junior High School Mathematics Education

The integration of deep learning in mathematics education refers to the use of artificial intelligence-based learning technology, intense learning neural networks, to support adaptive, personalised, and responsive learning processes for students (Ghayoumi, 2025; Bronstein et al., 2017). Deep learning is a sophisticated machine learning technique capable of analysing complex student learning data, detecting patterns, and providing real-time feedback tailored to each student's needs (Chollet et al., 2022; Li Fei-Fei & Perona, 2006).

The primary aspects comprise: firstly, the analysis of student behaviour through the utilisation of deep learning to identify specific difficulties (errors in algebra questions); secondly, the provision of adaptive feedback, enabling timely intervention through an intelligent tutoring system; and thirdly, the prediction of academic performance, facilitating the earlier identification of at-risk students by teachers (Cohrs et al., 2024; Higham & Higham, 2019). Indicators of the effectiveness of deep learning integration include: the accuracy of strategies in predicting performance, the speed and relevance of feedback provided, and the strengthening of the quantity and quality of learning interactions (duration of system use and frequency of use of adaptive learning tools) (He & Tao, 2025; Celledoni et al., 2021).

The integration of deep learning into junior high school mathematics education brings several significant advantages. First, personalised learning can tailor material to students' abilities and learning styles in real time. This means that both weaker and faster-developing students can receive challenges tailored to their needs, strengthening engagement and conceptual understanding (Ghosh et al., 2025; Rabe & Szegedy, 2021). Second, student-teacher collaboration is enhanced by intelligent interventions that make the learning process more interactive and motivating for students. Third, teacher time is optimised, and decisions are data-driven, as teachers are assisted by automatic feedback and performance analysis, allowing them to focus on critical guidance and advanced

learning strategies (Hasan et al., 2026; Al-Busaidi & Al-Syabi, 2021). Deep learning integration utilises learning analytics to adjust mathematics content dynamically (Zang et al., 2026; Cui & Teo, 2023).

2.4. Strengthening Critical Thinking in Deep Learning-Oriented Mathematics Education

Critical thinking is defined as an intellectual process that entails the ability to analyse, evaluate, and synthesise information to make sound decisions. Indicators of critical thinking are as follows: 1) Analytical skills, which involve the process of breaking down information into smaller parts in order to understand its structure; 2) Evaluation skills, which involve the process of assessing arguments based on accuracy and credibility; and 3) Inference skills, which involve the ability to draw logical conclusions from available data (English, 2023; Demir & Namdar, 2019).

Strengthening critical thinking in the context of deep learning-oriented mathematics learning refers to students' ability to reflectively and analytically understand, evaluate, and develop deep mathematical reasoning through learning that requires real problem solving and higher-order thinking strategies. Deep learning emphasises students' active involvement in exploration, inference, reflection, and metacognition rather than mere rote memorisation (Göhner & Krell, 2020; Dwyer et al., 2014).

The application of deep learning has been demonstrated to enhance students' critical thinking skills. The following aspects are conducive to critical thinking: interpretation, analysis, evaluation, inference, explanation, and self-regulation (Han et al., 2025; Amato et al., 2017). In this study, three critical thinking indicators were utilised, as previously referenced. In the context of mathematics education with a focus on deep learning, students are expected to engage in meticulous problem-solving, discerning the interconnections between concepts, evaluating arguments, deriving conclusions substantiated by evidence, elucidating their thought processes, and contemplating the strategies employed (Han et al., 2025; Amato et al., 2017). More specific studies in mathematics highlight the abilities to interpret, analyse, evaluate, explain, and even synthesise (Ozcanli & Baysal, 2022; Wang et al., 2022). Students must evaluate and articulate their own and others' thought processes, whilst concomitantly comprehending the interrelationships between concepts through a variety of representations (Cheng et al., 2021; Rahman et al., 2021).

The integration of critical thinking in deep learning-oriented mathematics has several advantages, namely a) strengthening conceptual understanding and transferability, b) fostering independence in reflection and metacognition among students, and c) strengthening students' readiness to face real contextual problems, not just routine questions (Zheng et al., 2020; Shishigu et al., 2019). By engaging in activities such as argument evaluation and self-reflection, students can select the most suitable strategies, question assumptions, and modify answers when necessary (Chen et al., 2025; Fang et al., 2023). is a pivotal skill in the 21st century (Jiang et al., 2020; Weng et al., 2020).

In the context of junior high school mathematics education, implementing deep learning methodologies that incorporate real-world context-based projects, computational thinking, and problem-solving exercises has been shown to foster critical thinking skills in students. This approach encourages students to engage in systematic problem analysis, develop solution algorithms, and evaluate results, thereby facilitating the cultivation of their critical thinking abilities (Leijen et al., 2021; Weintrop et al., 2016). Furthermore, computational thinking skills have been shown to be a strong predictor of mathematical critical thinking. Previous studies have shown a high correlation ($r \approx 0.897$, determination $\approx 80\%$) between students' computational skills and their mathematical critical thinking skills. This proves that structural and algorithmic problem-solving are important (Huang et al., 2022; Geisinger, 2016).

3. Methodology

This research is categorised as developmental qualitative research, aiming to explore how deep learning-oriented mathematics instruction can effectively enhance students' critical thinking skills. Qualitative research is particularly appropriate for investigating complex social processes because it enables researchers to capture participants' lived experiences, meanings, and contextual interpretations (Fridayanti, 2023; Sutama et al., 2022). The research adopts an ethnographic design, which is particularly appropriate for examining cultural patterns, social interactions, and behavioural routines within a specific community. Ethnographic research focuses on understanding values, norms, and practices from the emic perspective, enabling researchers to interpret social realities from participants' own viewpoints (Sutama et al., 2022; Reeves et al., 2020). A qualitative approach provides rich, descriptive data that reveal how teaching practices, classroom interactions, and cognitive engagements occur naturally in real school settings. Recent methodological literature also emphasises that qualitative inquiry is essential for understanding dynamic, contextual, and socially constructed learning phenomena (Sutama et al., 2024; Nowell et al., 2019).

The research site is located at SMP Negeri 3 Kartasura, Sukoharjo, Central Java. SMP Negeri 3 Kartasura has been awarded the prestigious A accreditation, a testament to its academic and managerial excellence. The school where the research is conducted operates in the morning and is located near Adi Soemarmo Airport. The institution's strategic location ensures accessibility for students from various areas around Kartasura. The research period is scheduled to commence in early February 2025 and conclude at the end of July 2025.

The data collected in this research comprises both primary and secondary data. Primary data were obtained through in-depth interviews with the school principal, mathematics teachers, and students of class VIII B, as well as through participatory observation during mathematics lessons to capture authentic classroom interactions. Secondary data were derived from document analysis involving mathematics textbooks, student record books, grade reports, teachers' daily logs, and archives of students' mathematical work (Sutama et al., 2022). In line with ethnographic principles, the researcher functions as the primary instrument of data collection and analysis. This role encompasses direct engagement in interviews and observations, the interpretation of data through the researcher's reflective understanding, and sustained immersion in the field to build rapport and generate contextualised insights. Researchers must also remain aware of their subjectivity, actively practising reflexivity to minimise bias and maintain analytical integrity (Sutama & Suci, 2024; Tracy, 2020). Furthermore, the flexible and adaptive nature of qualitative instruments enables the researcher to modify strategies in response to emerging field conditions, thereby enhancing the credibility of the findings (Nowell et al., 2019).

To ensure the validity and trustworthiness of the data, the study employs both source and method triangulation. Source triangulation involves comparing information obtained from various informants to verify consistency and strengthen the credibility of the interpretations. In contrast, method triangulation integrates multiple data collection techniques—such as interviews, observations, and document reviews—to enhance the reliability and robustness of the findings (Creswell & Poth, 2018; Flick, 2018). Data analysis was conducted using an inductive approach, wherein categories, patterns, and themes emerged naturally from the field data without imposing preconceived theoretical frameworks. Inductive analysis allows researchers to move from specific observations to broader conceptualisations through repeated cycles of reading, coding, categorising, and refining themes until stable patterns are identified (Sutama et al., 2023; Braun & Clarke, 2021). Throughout the analysis process, the researcher continuously interacted with the data, revising categories and interpreting meanings based on empirical field findings. The analysis followed a cyclical process involving data collection, data reduction, data display, verification, and conclusion drawing, enabling the development of grounded interpretations that align with the study's

ethnographic orientation (Fuadi et al., 2021). In this study, researchers continuously interacted with the data, modified categories, and interpreted information in accordance with field findings. The data analysis process is cyclical, involving data collection, data reduction, data presentation, verification, and conclusion drawing (see Figure 1).

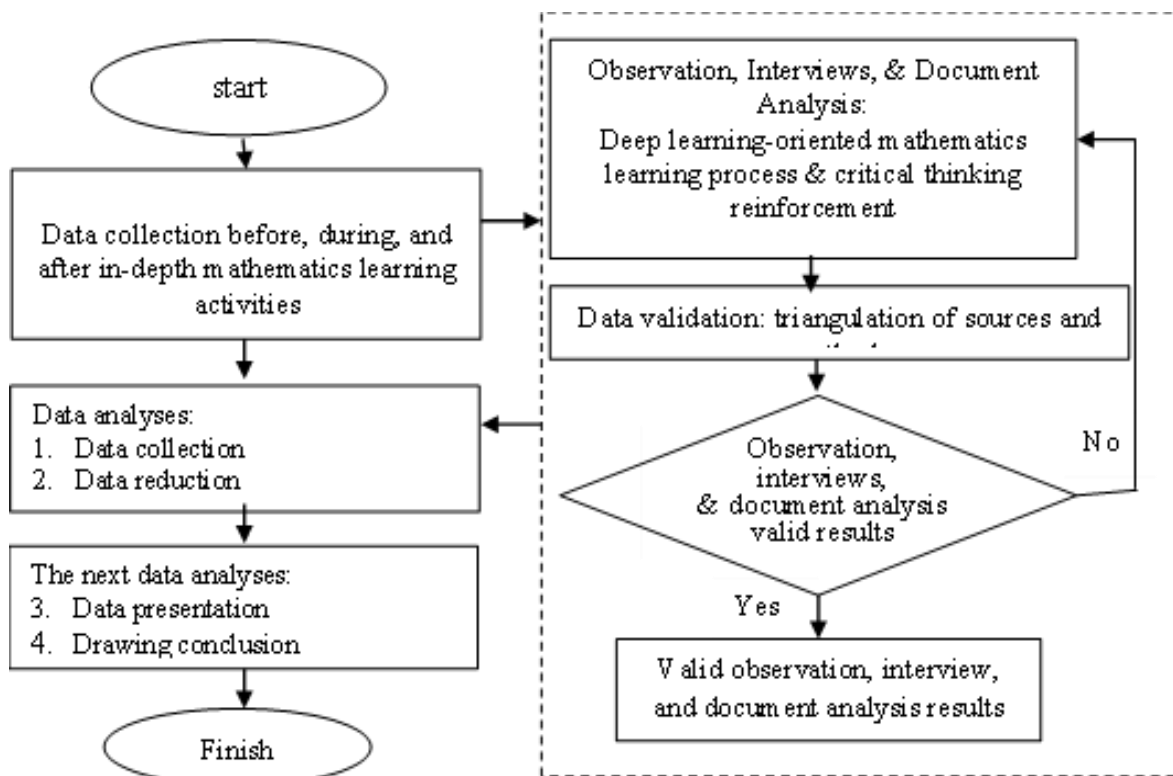


Figure 1. Data collection activities through to data analysis

4. Results

4.1. Deep Learning-Oriented Mathematics Learning Process in Junior High School

The deep learning-oriented mathematics learning design for strengthening critical thinking skills in junior high schools is presented in Figure 2. The learning design was developed by researchers based on preliminary data and related theories. The model design was then revised in Focus Group Discussions (FGD) in an open forum. The FGD participants were policymakers, users, experts, and researchers. The results of this learning model design are expected to effectively strengthen critical thinking, as reflected in indicators of students' analytical, evaluative, and inferential abilities.

The learning process comprises three distinct phases: preliminary activities, core activities, and closing activities. The programme's introductory activities encompass conditioning and apperception, pre-tests, and an analysis of learning objectives and the learning process. The following core activities are identified: a) the linkage of prior knowledge and real-world contexts, b) the exploration and discovery of concepts, c) the application of concepts in contextual problems, d) the reinforcement and transfer of concepts, and e) controlled and independent practice. The concluding activities encompass a reflective process and the formulation of conclusions, as well as post-tests, follow-up, and the programme's definitive conclusion. The following section presents a comprehensive overview of the mathematics learning model design oriented towards deep learning.

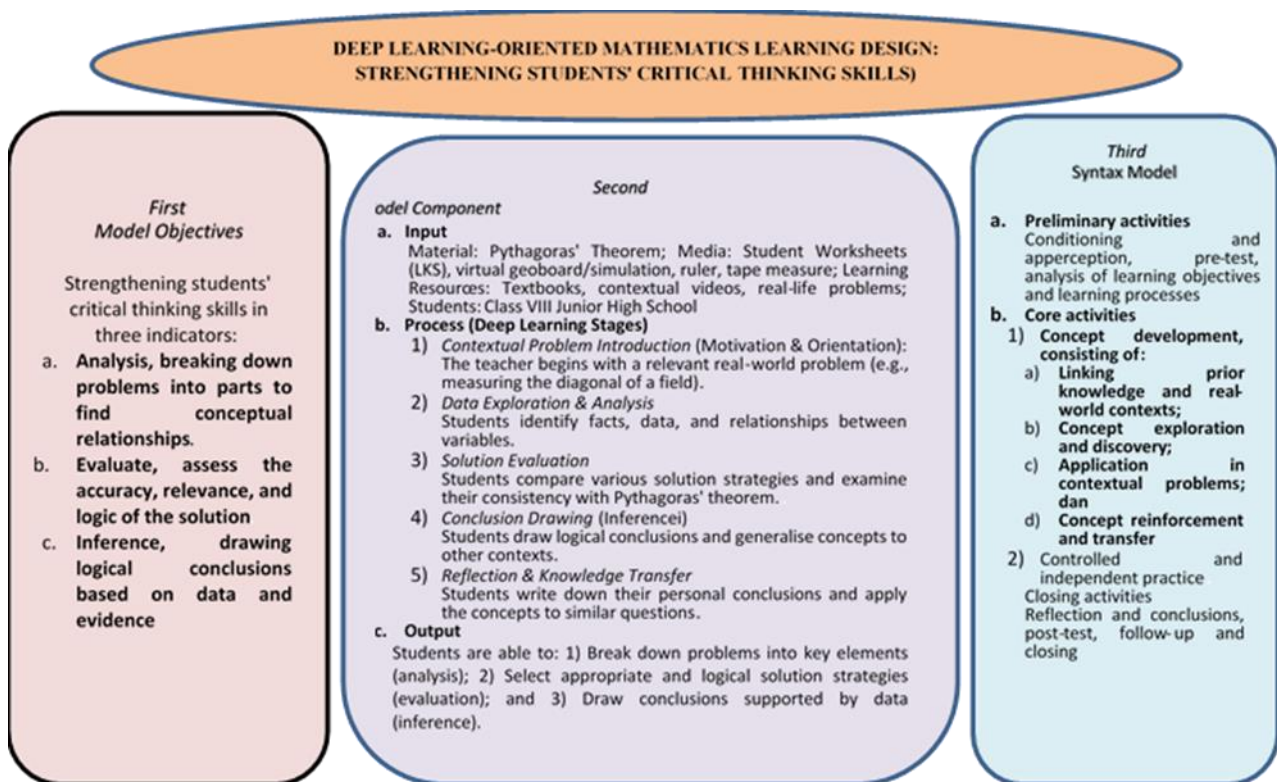


Figure 2. Deep Learning-Oriented Mathematics Learning Design to Strengthen Critical Thinking in Junior High School Students

Conditioning and apperception are essential in the introduction to deep learning-oriented mathematics learning. Conditioning involves preparing students mentally and emotionally, whether it be responding to greetings, preparing facilities and a comfortable learning environment, or praying together to begin learning. This process also involves apperception, which links the material to be learned with students' initial abilities in accordance with the real-world context. As an illustration, verbal apperception is as follows.

Teacher: " Children, today we are going to learn about the Pythagorean Theorem. Does anyone know what the Pythagorean Theorem is?"

Student 1: "I have heard of it, Miss. It is about right-angled triangles, right?"

Teacher: "Correct! The Pythagorean theorem applies to right-angled triangles. Now, does anyone know the formula?"

Student 2: "I do, Miss! The formula is $a^2 + b^2 = c^2$, where a and b are the perpendicular sides, and c is the hypotenuse."

Teacher: " Excellent! So, if we know two sides of a right-angled triangle, we can find the third side. For example, the first perpendicular side is 3 cm long, and the second is 4 cm long. How long is the hypotenuse?"

Student 3: "I will try, Miss! $c^2 = 3^2 + 4^2$, $c^2 = 3^2 + 4^2$... So, $c^2 = 9 + 16 = 25$. That means $c^2 = 25$ and $c = 5$ "

Teacher: " Good! Now, what if we know that the hypotenuse is 13 cm and one leg is 12 cm? How do we find the other leg?"

Student 4: "We use the same formula, Ma'am. $c^2 - a^2 = b^2$. So, $b^2 = 13^2 - 12^2 = 169 - 144 = 25$. That means $b^2 = 25$ and $b = 5$ "

Teacher: " Correct! Children, you now understand how to use Pythagoras' theorem to solve problems. Does anyone have any questions?"

Student 5: " Miss, if the shape is not a right-angled triangle, can we still use Pythagoras' theorem?"

Teacher: " Good question! Pythagoras' theorem only applies to right-angled triangles. For other triangles, there are other rules, such as the cosine rule. We will learn about that later in a higher class."

Pre-test serves to identify prior knowledge, measure readiness, and map students' gaps in understanding the material before learning begins. An alternative oral pre-test conducted by the teacher at the research site, with the question: 'A triangle has sides measuring 6 cm and 8 cm. Is this triangle a right-angled triangle? Explain your reasoning.' The teacher then asked the following questions orally. (T: Teacher & S: Student i).

T: Who can solve the Pythagorean Theorem formula for a right-angled triangle with sides a , b , and hypotenuse c ?

S6: I can, Miss, it is $c^2 = a^2 + b^2$

T: Very well. Next, if $a = 6$ cm, $b = 8$ cm, what is c ? Who can solve it?

S7: I can, Miss. From that formula, $c = 10$

T: Good, that means the Pythagorean Theorem is satisfied. So what is your conclusion?

S8: The triangle is a right-angled triangle because the lengths of its sides satisfy the Pythagorean Theorem.

Analysis of deep learning-oriented mathematics learning objectives is useful for designing relevant and focused learning, ensuring that each learning activity supports the development of critical thinking. In this analysis of objectives, teachers present their objectives (as mentioned above), and students examine them and ask questions if anything is unclear. Analysis of learning objectives helps to ensure that the learning process is effective, focused, and tailored to the needs of students in the digital age.

Learning process analysis refers to efforts to evaluate and design learning steps that can motivate students to analyse, evaluate, and solve problems logically and systematically. In this context, it involves reviewing the stages of learning, the strategies used, and the interaction between mathematical material and the application of deep learning principles. This process aims to provide challenging experiences and strengthen students' critical thinking skills in a structured and applicable manner.

The core activity, deep learning-oriented mathematics learning, on the Pythagorean Theorem material in Grade VIII Junior High School, was first related to concept development. In developing concepts by linking prior knowledge and authentic contexts, the teacher displayed a picture of a real situation, 'a ladder leaning against a wall'. Then, they ask provoking questions, such as, 'If we know the lengths of two sides of a right-angled triangle, how can we find the unknown side?' 'Why don't construction workers measure the diagonal directly?' Students are asked to make assumptions (hypotheses) based on their prior knowledge of triangles and the area of squares.

Concept development is related to exploration and discovery of concepts. Students are divided into small groups (3–4 people) and given Student Worksheets (SW). a) Draw a right-angled triangle with different sides (e.g. 3 cm, 4 cm). b) Make a square on each side of the triangle. c) Calculate the area of the squares on the two perpendicular sides and on the hypotenuse. d) Compare the results and find the pattern of the relationship. The teacher guides the students to realise the relationship between the square of the hypotenuse and the sum of the squares of the other two sides.

Concept development related to application in contextual problems involves giving students story problems that are relevant to everyday life. a) Determine the length of the ladder needed to

reach a window 4 m above the ground, with the foot of the ladder 3 m from the wall. b) Determine the diagonal of a 24 m × 7 m field. Students solve problems by writing down the calculation steps and logical reasoning. Teachers emphasise that deep learning requires reasoning, not just formula substitution.

Concept development related to concept reinforcement and transfer, students were asked to create their own Pythagoras questions from their school/home environment. Groups exchanged questions and tried to solve them. Teachers provided feedback, emphasising the connection between Pythagoras' concept and other fields (physics, engineering, design).

The development of concepts related to controlled and independent practice involves students practising the application of Pythagoras' theorem with guidance from the teacher and working on more complex problems independently to reinforce their understanding of the concept and test their comprehension. Controlled practice problem: a ladder is leaning against a wall. The bottom end of the ladder is 6 m from the foot of the wall, while the length of the ladder is 10 m. Determine the height of the wall reached by the top end of the ladder! (Use the image and discuss the steps). Independent training problem: a farmer has a piece of land in the shape of a right-angled triangle. The lengths of the two sides are 8 m and 15 m. He wants to put up a fence around the plot. Determine a) the length of the sloping side of the plot, b) the perimeter of the entire plot, and c) if the price of the fence is Rp 50,000 per metre, how much will the total cost be?

Closing activities for deep learning-oriented mathematics learning to strengthen critical thinking, in reflection activities, through question and answer sessions, students convey things related to what they have mastered, what they have not mastered, the root causes of not mastering, and alternative learning solutions. Conclusion-drawing activities, also through question and answer sessions, guided students in summarising the material that had been learned from the beginning to the end of the core activity. In the reflection and conclusion-drawing activities, students wrote in their learning journals about the material that had been learned as follows.

1 Summary of key concepts

- a. The definition of Pythagoras' theorem should be written down in your own words.
- b. The following formula should be noted: In the context of a right-angled triangle, the length of the sides (a, b) and the hypotenuse (c) can be calculated as follows: $a^2 + b^2 = c^2$.

2 Problem-solving steps

The process of problem-solving should be documented, with each step clearly delineated, commencing with the identification of the hypotenuse and culminating in the final calculation.

3 Sample questions and solutions

Copy sample questions that are considered most helpful in understanding the concept, along with complete steps and answers.

4 New findings and insights

Writing down new things learned that day, 'It turns out that the Pythagorean Theorem only applies to right-angled triangles.' 'The length of the hypotenuse is always greater than the other sides.'

5 Self-reflection

Write down the sections that have been understood and those that have not been understood to help teachers identify further needs.

6 Learning conclusions

Summarising the key points of the learning material, 'Pythagoras' theorem can be used to calculate the length of the sides of a right-angled triangle if the other two sides are known.'

7 Practical applications

Write at least one example of application in everyday life, 'Calculating the length of stairs needed to reach the roof.'

Post-tests are conducted orally or in writing. Oral tests are conducted if the subject matter has not been fully covered, while written post-tests are conducted if the subject matter has been fully covered. Below are examples of oral and written post-test questions and answers from one of the study participants.

Oral question: A student measures the distance from point A to point C via point B. The distance AB is 9 metres, and the distance BC is 12 metres. These two segments form a right angle at point B. Determine the direct distance from point A to point C without passing through point B. Explain the steps!

Jawab : Identifikasi bentuk segitiga:
 Segitiga ABC merupakan segitiga siku-siku di B,
 sehingga berlaku Teorema Pythagoras: $AC^2 = AB^2 + BC^2$
 Substitusi nilai: $AC^2 = 9^2 + 12^2$
 $AC^2 = 81 + 144$
 $AC^2 = 225$
 Akar kuadrat
 $AC = \sqrt{225} = 15 \text{ m}$
 Jadi, jarak langsung A ke C adalah 15 meter

Figure 3. Subject 5 Answers to Oral Post-test Questions

Written question: Students are presented with the following problem: 'A student wants to make a sloping board to lower goods from a lorry to the ground. The height of the lorry bed from the ground is 1.2 metres. The horizontal distance from the lorry to the end of the board on the ground is 2 metres. Determine the length of the board required so that it can be placed from the lorry bed to the ground precisely.'

Jawab : Identifikasi segitiga: Tinggi bak truk = 1,2 m → sisi tegak (a)
 Jarak mendatar = 2 m → sisi alas (b)
 Panjang papan = sisi miring (c)
 Teorema Pythagoras: $c^2 = a^2 + b^2$
 $c^2 = (1,2)^2 + (2)^2$
 $c^2 = 1,44 + 4$
 $c^2 = 5,44$
 Panjang papan : $c = \sqrt{5,44}$
 $c = 2,33 \text{ m}$
 Jadi, panjang papan yang diperlukan adalah ± 2,33 meter

Figure 4. Subject 9 Answers to Written Post-test Questions

Follow-up and closing activities in deep learning-oriented mathematics learning aim to ensure that students can relate newly learned concepts to real-world applications and encourage them to learn the following teaching material. Follow-up activities include a) feedback and clarification, where teachers respond to students' reflections, correct misconceptions, and appreciate creative answers. b) Mini projects (project assignments), where students are asked to create illustrated reports on the application of the Pythagorean Theorem in their home environment. c) Connections to subsequent material, where teachers explain that the right-angled triangle concepts that have been learned will form the basis for the following material, namely Trigonometric Ratios in Right-Angled Triangles.

The teacher will then move on to the closing activities. These will include a) An emphasis on the importance of deep learning, which involves understanding concepts in depth, connecting them, and applying them in various situations. b) The provision of motivation through the statement that mathematics is not just about calculating, but about understanding patterns in life. c) A closing greeting from the teacher, followed by a response from the students.

4.2. Strategies for strengthening students' critical thinking in deep learning-oriented mathematics education in junior high schools

Strategies for strengthening students' critical thinking, namely in terms of the following indicators: 1) analytical skills; 2) evaluation skills; and 3) inference skills. Strategies for strengthening critical thinking and analytical skills in deep learning-oriented mathematics were implemented at the research site through the following steps.

1 Problem Identification: the teacher guides students to identify problems in questions by understanding the relationships between elements (sides of triangles, right angles, etc.);

2 Information breakdown: students divide the problem into smaller parts, namely 1) Determining which triangle contains a right angle; 2) Determining the known sides and the unknown sides; and 3) Selecting the appropriate Pythagorean theorem.

3 Strategy selection: students choose a solution strategy based on the information provided (using $a^2 + b^2 = c^2$);

4 Application of concepts: students apply Pythagoras' concept to contextual problems, not just simple calculations, and

5 Evaluation and reflection: students review their results and explain the reasons behind the steps they took. The following are practice questions for strategies to strengthen students' analytical skills.

A farmer wants to install a fence forming a right-angled triangle in the corner of his field. One side is 9 metres long, and the other side is 12 metres long. He wants to know the length of the hypotenuse so that he can buy the right length of wire fencing. 1) Determine the length of the hypotenuse! 2) If the price of fencing wire is Rp 25.000 per metre, calculate the total cost of fencing wire for the hypotenuse alone!

Jawab : Identifikasi masalah :

Segitiga siku-siku dengan sisi tegak 9 m dan 12 m, dicari sisi miring

Gunakan Teorema Pythagoras:

$$c^2 = a^2 + b^2$$

$$c^2 = 9^2 + 12^2$$

$$c^2 = 81 + 144$$

$$c^2 = 225$$

$$c = \sqrt{225} = 15 \text{ m}$$

Hitung biaya kawat pagar:

$$\text{Biaya} = 15 \times 25.000 = \text{Rp } 375.000,00$$

Jadi, 1) panjang sisi miring = 15 meter

2) biaya kawat pagar untuk sisi miring = Rp 375.000,00

Figure 5. Subject 12 Answer to Question on Strengthening Analytical Skills

The strategy for strengthening critical thinking for evaluation ability indicators in deep learning-oriented mathematics learning was implemented at the research site with the following steps:

1 The teacher then contextualizes the problem by introducing a relevant real-world problem, such as calculating the length of a ladder leaning against a wall. The problem is presented with two or more different solutions (some of which are correct and some of which are not) to prompt evaluation

2 In order to elicit cognitive conflict, the teacher presents two divergent answers to the same problem and invites students to evaluate which is correct and provide the rationale for their assessment. Students are tasked with identifying procedural or logical errors in the incorrect answer.

3 In the context of a small group discussion (Deep Exploration), students are tasked with working collaboratively in groups to analyse the steps of the solution and indicate the incorrect parts. The teacher fulfils a facilitative role, posing questions that serve as catalysts for deeper thinking, such as 'Are all the conditions for a right-angled triangle fulfilled?' and 'Is the relationship between the squares of the sides correct?'

4 In the verification stage of the process, pupils are required to verify the correct answer by applying the Pythagorean Theorem formula in both a complete and systematic manner. Pupils are tasked with comparing calculation results with real-world logic, that is, whether the results are logical within the given context.

5 It is imperative that students engage in critical reflection, which involves the formulation of evaluative conclusions that articulate the merits and shortcomings of a given method. This process entails identifying lessons learned that can inform future practices and prevent the recurrence of past errors. Teachers provide feedback based on reasoning, not on whether an answer is right or wrong. The following practice questions are designed to help students hone their evaluation skills.

A ladder is positioned perpendicular to a vertical surface, with a distance of 6 meters separating the lowermost rung from the surface. The height of the wall, measured from the top of the ladder, was 8 meters. The following steps are to be followed by the student in order to calculate the length of the ladder:

1. $c = 6 + 8 = 14$
2. The staircase is 14 metres long.

Evaluate the student's solution. Is the answer correct? Explain your reasoning and provide the correct calculation.

Langkah siswa tersebut salah karena menjumlahkan langsung pangang kaki segitiga
 Teorema Pythagoras menyatakan bahwa :
 $c^2 = a^2 + b^2$, dimana c sisi miring (pangang tangga), dan a, b sisi tegak lurus segitiga.
 Sehingga perhitungan yang benar:

$$c^2 = a^2 + b^2$$

$$c^2 = 6^2 + 8^2$$

$$c^2 = 36 + 64$$

$$c^2 = 100$$

$$c = \sqrt{100}$$

$$= 10 \text{ meter}$$

Figure 6. Subject 22 Answers to Questions on Strengthening Evaluation Skills

The strategy to strengthen critical thinking for the inference ability indicator in deep-learning-oriented mathematics learning was implemented at the research site through the following steps.

1 Contextualising the problem, the teacher begins the lesson by presenting a real-life problem (the slant height of a staircase) that requires the application of Pythagoras' theorem. The aim is to motivate students to observe the initial data and predict the relationship between variables.

2 Data and fact exploration: pupils work in groups to identify known and unknown information about the problem. Pupils record the triangle's sides, take measurements, or use the provided data.

3 Hypothesis formulation: students are asked to make an initial guess (preliminary inference) about the solution to the problem before performing calculations. "If the slanted side is longer, then the other side must satisfy $a^2 + b^2 = c^2$."

4 Using evidence and calculations, students use the Pythagorean Theorem to prove the truth of the hypothesis that has been proposed. Students focus on connecting facts (numerical data) with logical conclusions.

5 Reflection and generalization: students conclude and relate the results to other similar problems, thereby generalizing the Pythagorean principle to various contexts. The following are practice questions for strengthening students' inference skills.

A cleaner wants to place a ladder to clean the school windows, which are 4 meters above the ground. He places the foot of the ladder 1.5 meters from the wall. Is the length of the ladder at least 4.5 meters? Use Pythagoras' theorem to conclude.

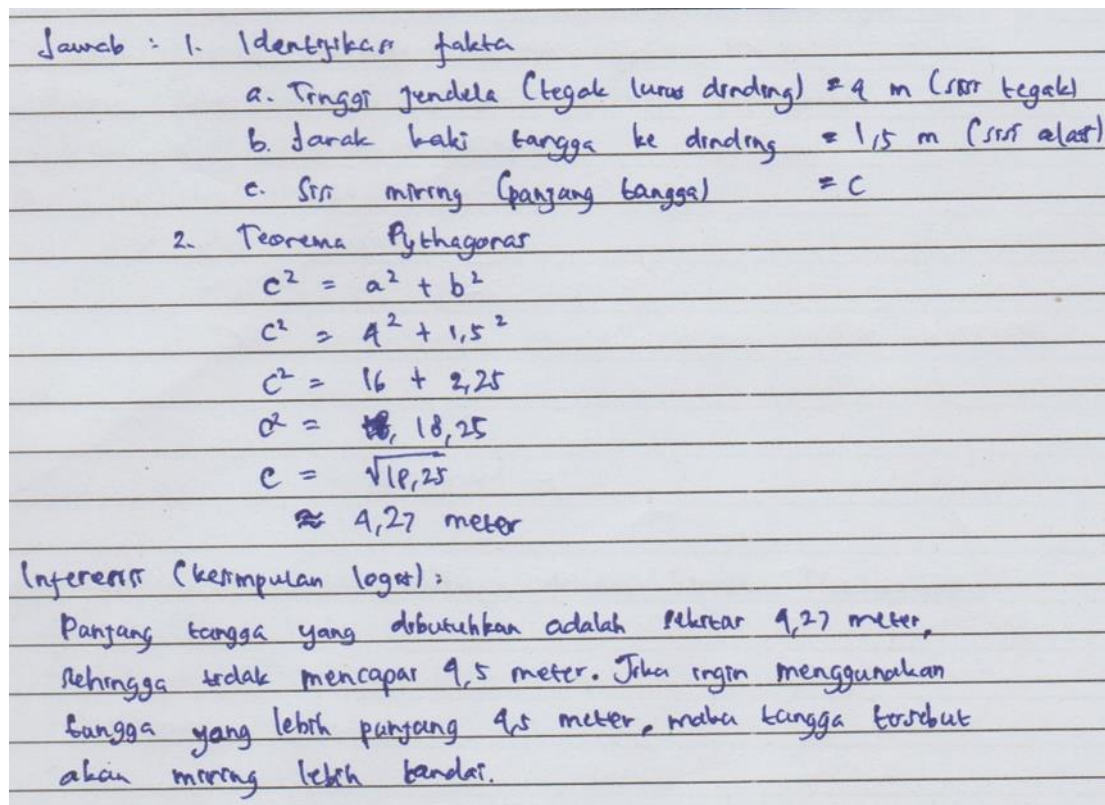


Figure 7. Subject Answers 25 Questions on Strengthening Inference Skills

5. Discussion

5.1. Deep Learning-Oriented Mathematics Learning Process in Junior High School

The deep learning-oriented mathematics learning process in the preliminary activities and in student activities in conditioning and apperception was very enthusiastic and tended to involve all students actively. The results of this study are supported by previous studies, which state that conditioning and apperception are essential in mathematics learning (Weng et al., 2020; Al-Qatf et al., 2018). Other research results also indicate that preparing comfortable learning facilities and environments, relating the material to be discussed to students' prior knowledge, and applying real-world contexts are very important for student engagement in learning (Berner et al., 2022; Geisinger, 2016). This means that conditioning and apperception in deep-learning-oriented mathematics learning are essential for students to be independent, enthusiastic, and active learners.

The preliminary activities demonstrated strong student enthusiasm and active participation during conditioning and apperception, both of which play a crucial role in deep-learning-oriented mathematics instruction. These findings support recent studies that emphasize the importance of well-structured introductory activities in increasing student readiness, reducing cognitive load, and enhancing willingness to engage in higher-order thinking (Choudhary et al., 2022; Zheng et al., 2020). Similarly, preparing a comfortable learning environment and connecting new material to students' prior knowledge are essential for achieving meaningful engagement, corroborating evidence that contextualization increases motivation and conceptual accessibility (Dahal & Shakya, 2023; Shishigu et al., 2019). Thus, conditioning and apperception contribute significantly to the cultivation of independence, enthusiasm, and active learning behavior.

Apperception activities in this study presented students with contextual mathematical problems designed to stimulate analytical, evaluative, and integrative reasoning. This finding aligns with recent research indicating that meaningful apperception tasks enhance cognitive engagement and foster critical thinking through opportunities for exploration, argumentation, and reflection (Cevikbas et al.,

2024; Lestari et al., 2024). The results also resonate with studies showing that introducing lessons with contextual questions enables students to identify patterns, reflect on prior knowledge, and apply reasoning to new situations—key processes that support logical, evidence-based decision-making (Nikmah et al., 2024; Siahaan et al., 2024). Therefore, the role of apperception in deep learning transcends mere motivation; it actively nurtures critical and reflective thinking habits

The use of pre-tests in the preliminary activities proved effective in diagnosing students' prior understanding and readiness for deep learning. The findings show that pre-tests guide teachers in designing instruction tailored to student needs, consistent with recent research emphasizing that diagnostic assessment informs differentiated instruction and improves learning effectiveness (Guggemos & Seufert, 2021; Fung, 2017). Pre-tests also revealed students' initial capabilities in deep thinking, reinforcing results from studies indicating that early measurement of analytical and critical reasoning supports the development of higher-order thinking throughout mathematics learning (Dalehefte & Canrinus, 2023; Korkmaz et al., 2019). Thus, pre-tests function not only as assessment tools but as essential components of strategic instructional planning.

The analysis of learning objectives in this research served to ensure that each learning activity supported the development of students' critical thinking in a focused and systematic manner. This is strongly supported by recent findings that rigorous learning-objective analysis helps teachers identify core competencies, align instruction with cognitive outcomes, and integrate critical-thinking elements into mathematics lessons (Duan, 2022; Shute et al., 2017). Additional research likewise highlights that clear objectives enhance instructional coherence and ensure that learning activities are relevant to students' needs in the digital era, where analytical reasoning skills are increasingly essential (Lhiadi et al., 2021; Hermes & Rimanoczy, 2018). The analysis of learning objectives plays a pivotal role in ensuring the efficacy, focus, and alignment of the learning process with students' needs in the digital age.

The analysis of the learning process further reveals that deep learning strategies offer structured, meaningful, and context-rich experiences that encourage critical thinking. These results are consistent with recent research demonstrating that deep learning approaches in mathematics strengthen students' analytical, evaluative, and problem-solving skills, equipping them with competencies essential for navigating complex mathematical tasks (Kovač et al., 2023; Yang et al., 2022). Moreover, several studies support the conclusion that deep mathematics learning fosters rational reasoning and reflective judgment, both of which are fundamental for student success in the modern era (Dai et al., 2023; Qu et al., 2021). This means that the mathematics learning process that develops critical thinking enables students to solve problems rationally and participate in the modern era.

In the core learning activities, the development of mathematical concepts was facilitated through collaborative exploration, in which students discovered formulas and definitions with teacher guidance. This process reflects a constructivist approach, in which students build meaning through interaction and dialogue. Recent studies affirm that collaborative learning deepens mathematical understanding by enabling students to articulate their thinking, test ideas, and negotiate meaning collectively (Paleenud et al., 2024; Kim & Jang, 2023). Additional evidence shows that collaborative problem-solving strengthens the acquisition and application of mathematical concepts, particularly when students are required to justify and evaluate one another's reasoning (Liu, 2022; Darling-Hammond et al., 2019). This means that concept development in deep mathematics learning can build meaning related to the mathematical concepts studied in depth through four student activities.

Concept development consists of four student activities, namely 1) linking prior knowledge and real-world contexts, 2) exploring and discovering concepts, 3) applying concepts to contextual

problems, and 4) reinforcing and transferring concepts. Students are presented with the following problem: "Draw a right-angled triangle with sides a and b , and hypotenuse c . Figure 8.2 is a simple illustration of finding the Pythagorean theorem in a right-angled triangle:

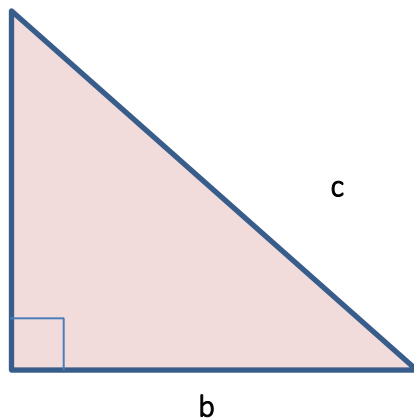


Figure 8.1. Right-Angled Triangle

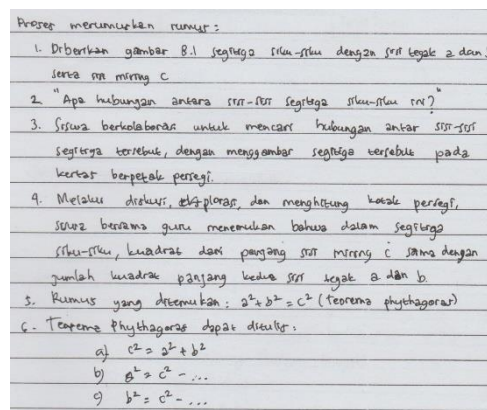


Figure 8.2. The Discovery of Pythagoras' Formula

Controlled exercises in core mathematics learning activities oriented towards deep learning, students work in groups to solve HOTS (Higher Order Thinking Skills) type questions. This activity challenges students not only to remember information but also to analyze, evaluate, and apply knowledge in more complex situations. Previous research has shown that HOTS-based learning can strengthen students' problem-solving skills (Kunang et al., 2024). Similarly, previous studies have shown that controlled exercises that incorporate group discussions help students think more critically and deeply about mathematical concepts (Opesemowo & Ndlovu, 2024; Siagian et al., 2019). This means that teacher-controlled group exercises can help students think critically about deep mathematical concepts.

In the independent practice phase, students completed problems of varying difficulty (easy, medium, difficult), including HOTS items, at their own pace. This design supports differentiated cognitive engagement and aligns with research showing that self-paced problem solving tailored to students' cognitive readiness fosters confidence and strengthens critical thinking (Zhao et al., 2022). Other research findings indicate that independent exercises, especially HOTS questions, are effective in strengthening students' critical thinking (Al-Doori et al., 2021). Moreover, this independent yet scaffolded work supports metacognitive development, particularly when students apply self-regulated strategies (monitoring, planning, evaluating), as documented in metacognitive learning research.

Activities in reflection and drawing conclusions in closing activities for mathematics learning, oriented towards deep learning, involve students writing in their learning journals about the material they have studied. This is supported by previous research findings, which state that reflection activities can train critical thinking, honesty, and creativity in learning (Cevikbas & Kaiser, 2020; Higham & Higham, 2019). Other research results indicate that concluding can foster discipline and creativity in learning (Mittal et al., 2022). Thus, reflection and conclusion-drawing activities in mathematics learning are essential to encourage students to be independent, honest, disciplined, critical thinkers, and creative.

Post-tests in the closing activities of mathematics learning oriented towards deep learning were conducted orally or in writing as previously stated. The results of this study are supported by previous studies that emphasise that structured oral questioning is used to monitor misconceptions about the

relationship between the sides of a square and the application of Pythagoras' triple while learning is still ongoing (Dai et al., 2023; Leenknecht et al., 2019). Meanwhile, other studies report that the use of HOTS-based written post-tests can strengthen formative-summative continuity, maintain the validity of learning decisions (remedial/enrichment), and remain compatible with deep learning-oriented learning (Campbell et al., 2019). This means that post-tests should be conducted both orally and in writing to maintain the continuity of learning between initial and subsequent material.

The following activities are to be undertaken in order to consolidate the deep learning-oriented mathematics learning: 1) feedback and clarification, 2) mini projects, and 3) linking to the following material. Previous research has shown that structured formative feedback strengthens students' conceptual understanding and metacognitive abilities (Lhiadi et al., 2021; Fauskanger & Bjuland, 2018). The extant research results on mini-project learning demonstrate that there is an improvement in mathematical concept understanding (Orhani, 2024; Mishra et al., 2021). The outcomes of this subsequent activity can consolidate students' understanding of concepts and foster their critical thinking.

For the concluding activity, the teacher ended the session by 1) emphasising the meaning of deep learning, 2) providing motivation, and 3) saying a closing greeting. These findings are in line with the results of a study (Panadero et al., 2021), which emphasised the importance of concluding activities to strengthen students' conceptual understanding and instill the meaning of deep learning. Moreover, research (Sabah et al., 2023) corroborates the efficacy of closing activities accompanied by reflection and motivation in enhancing students' retention of mathematical concepts. This study makes a novel contribution by emphasising the importance of closing activities that are not only formal but also meaningful, reflective, and motivating for students to learn further.

5.2. Strategies for strengthening students' critical thinking in mathematics learning oriented towards deep learning in junior high school

Strategies for strengthening critical thinking in analytical ability indicators, namely a) problem identification, b) information analysis, c) strategy selection, d) concept application, and e) evaluation and reflection. Problem-based learning effectively develops students' analytical skills by providing real-life tasks (Sen, 2022). Realistic mathematics learning has been proven to strengthen analysis (Suglo, 2024) significantly. Self-explanation and explanation-to-others strategies have been shown to strengthen analysis in mathematics and statistics (Destino et al., 2019). The application of the mathematical investigation approach provides greater analytical reinforcement than expository learning (Gabriel et al., 2017). This means that the strategy for strengthening analytical skills involves analysing, comparing, and assessing information in depth to find logical connections in solving real-life mathematical problems.

To answer the question in Figure 9.2, students are required to calculate the diagonal result and compare it with two alternative paths. This pattern is consistent with findings from meta-analyses, which confirm the effectiveness of diverse strategies and the flexibility of reasoning in mathematics in fostering critical thinking, rather than a single procedure (Sutama et al., 2022; Winje & Løndal, 2020). In addition, problem-based learning consistently encourages students to analyse real-world contexts. Active-collaborative learning that integrates modelling activities and reasoned tasks has been shown to strengthen analytical processes through comparing representations and strategies before making decisions (Fong et al., 2017). This means that the question encourages analytical thinking by having students not stop at calculating the diagonal results but also examine and compare two alternative paths to determine the most efficient and logical choice.

Question:

A student stands at one end of a rectangular field measuring 24 meters by 7 meters. He wants to walk diagonally from one corner to the opposite corner.

1. Determine the length of the diagonal path traveled by the student using the Pythagorean theorem.
2. Analyze why the diagonal path is more efficient than if the students walked along the long and short sides of the field

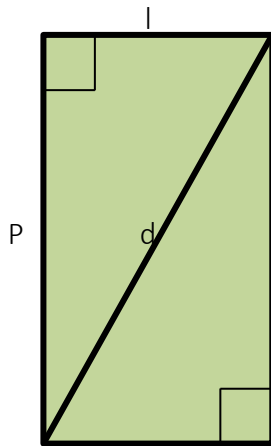


Figure 9.1. Field

1. Panjang diagonal d dapat dihitung dengan Teorema Pythagoras:

$$d = \sqrt{p^2 + l^2}$$

$$= \sqrt{(24^2 + 7^2)}$$

$$= \sqrt{576 + 49}$$

$$= \sqrt{625}$$

$$= 25 \text{ meter}$$

2. Jika siswa berjalan mengikuti sisi panjang dan lebar lapangan, maka jaraknya $= p + l = 24 + 7 = 31$ meter. Dengan berjalan diagonal, siswa hanya menempuh 25 meter, sehingga lebih efisien 6 meter.

Figure 9.2. Subject 27 Answers to Strengthening Analysis Questions

Strategies for strengthening critical thinking in evaluation ability indicators, namely a) contextualization of problems, b) cognitive conflict triggers, c) small group discussions, d) re-proofing, and e) critical reflection. Contextualizing problems strengthens evaluation skills by presenting relevant real-life situations (Zhao et al., 2025). The continuous application of small-group discussions, re-proving, and critical reflection facilitates students' conscious evaluation of the effectiveness and validity of solutions through social interaction, idea negotiation, and deep thinking (Li et al., 2024). Collaborative learning with reflective scaffolding significantly strengthens evaluation skills (Navpreet et al., 2025; Le, 2018). Thus, strategies to strengthen evaluation skills include learning activities that encourage students to assess, compare, and determine the validity of a solution based on evidence, logic, and relevant context.

In answering the questions in Figure 10.2, students not only select answers but also test, compare, and assess the validity of arguments before concluding the correct solution. This pattern aligns with PBL research findings and with metacognitive strategies that facilitate students' processes of testing hypotheses and evaluating evidence (Peng et al., 2024). Collaborative interaction, group discussions, and peer feedback strengthen the ability to assess the validity of arguments through the negotiation of evidence and the clarification of assumptions (Lin et al., 2023). Previous research also confirms that combining cognitive conflict, contextual tasks, and repeated reflection significantly strengthens evaluation skills compared to expository skills (Hattie & Donoghue, 2016). This means that the strengthening of critical thinking in evaluation skills is evident when students do not simply choose answers but also deeply examine, compare, and assess the validity of arguments before drawing correct conclusions.

Question:

A park officer wants to install lights on poles located at the corners of a right-angled triangular field. The sides of the triangle are 9 meters and 12 meters. Two students give different answers regarding the length of cable needed to draw electricity from the end of the sloping side to the pole.

Student (1) says the length of the cable is $(9^2 + 12^2) = 15$ meters. Student (2) says the length of the cable = $9 + 12 = 21$ meters.

Determine who is correct using Pythagoras' theorem.

Provide a critical evaluation of why the incorrect answer appears, and how students should assess the validity of their answers.

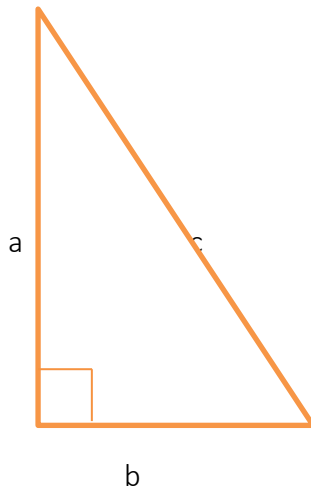


Figure 10.1. Right-Angled Triangle

1. Panjang kabel mengikuti sisi miring segitiga siku-siku dengan panjang sisi 9 m dan 12 m. Sehingga

$$c = \sqrt{a^2 + b^2}$$

$$= \sqrt{9^2 + 12^2}$$

$$= 15 \text{ m}$$

Jadi, siswa (1) benar, panjang kabel yang diperlukan adalah 15 meter.

2. Evaluasi kritis

a. Kesalahan siswa (2), karena ia menjumlahkan sisi tegak dan sisi datar tanpa mempersembahkan konsep segitiga siku-siku.

b. Protes evaluasi diperlukan untuk menilai keabsahan cara yang digunakan sesuai dengan kaidah Teorema Pythagoras atau hanya penjumlahan biasa.

c. Dengan mengacu kembali rumus, membandingkan dan refleksi konteks segitiga siku-siku, siswa dapat menyimpulkan bahwa cara yang benar menggunakan kuadrat sisi, bukan penjumlahan langsung.

Figure 10.2. Subject 3 Answers Strengthening Evaluation Questions

Strategies for strengthening critical thinking in inference ability indicators, namely a) contextualization of problems, b) exploration of data and facts, c) hypothesis formulation, d) use of evidence and calculations, and e) reflection and generalization. Research results show that providing authentic contexts encourages students to make logical inferences (Zhao et al., 2025; Hrin et al., 2017). Research results show that presenting contextual problems encourages students to make more meaningful inferences (Li et al., 2024; Amato et al., 2017). The skills of exploring data and formulating hypotheses have been shown to strengthen the ability to draw conclusions (Li et al., 2024; Afan et al., 2021). The use of empirical evidence, reflection, and generalization plays an important role in shaping students' inference skills (Huang et al., 2023; Wang et al., 2022). Thus, integrated strategies can strengthen students' ability to draw logical, consistent, and data-based conclusions.

The meaning of the questions and answers in Figure 7 is to train students to use critical thinking strategies in inference ability indicators, namely by connecting real contexts, exploring data (4 m high and 1.5 m distance), performing calculations using the Pythagorean Theorem, and then drawing logical conclusions as to whether the length of the ladder meets the minimum requirement of 4.5 metres or not. This process aligns with research showing that using contextual problems in mathematics encourages students to develop inference skills by linking formal concepts to real-life situations (Ran et al., 2025; Weisleder et al., 2019). In addition, data exploration and the use of calculated evidence have been shown to be effective in improving students' ability to draw rational conclusions (Zhang et al., 2024; Hoyt et al., 2019).

Furthermore, the results of this study are also supported by other previous studies. Previous studies have confirmed that the use of empirical evidence through mathematical calculations, followed by critical reflection, helps students develop their generalization skills to draw valid conclusions (Shin et al., 2025; Martins & Martinho, 2021). Previous research on reflection and generalization conducted after finding solutions reinforces deeper critical-thinking skills (Liu et al.,

2025; Gheisari et al., 2023). Thus, these questions and answers not only test students' procedural abilities but also foster critical thinking skills in drawing evidence-based inferences.

6. Conclusion

The deep learning-oriented mathematics learning process for strengthening critical thinking in junior high school students is related to three main aspects: 1) objectives, 2) components, and 3) learning syntax. The learning objectives are to strengthen students' critical thinking skills in analysis, evaluation, and inference. The learning components can be categorized as follows: (a) input, (b) process, and (c) output. The learning syntax comprises the following three stages: a) introductory activities, conditioning and apperception, pre-test, analysis of learning objectives and learning process; b) core activities, linking prior knowledge and real contexts; exploration and discovery of concepts, application in contextual problems, reinforcement and transfer of concepts, and controlled and independent practice; and c) closing activities, reflection and conclusion, post-test, follow-up and closing.

This paper outlines strategies to enhance students' critical thinking skills within a deep-learning-oriented mathematics education in junior high school. The strategies are designed to target and strengthen three key indicators of critical thinking. Firstly, strategies for strengthening critical thinking in the domain of analytical skills are presented. These strategies encompass the in-depth analysis, comparison, and assessment of information to ascertain logical connections in the resolution of real-life mathematical problems. Secondly, strategies for strengthening critical thinking in the evaluation skills indicator, namely learning activities that encourage students to assess, compare, and determine the validity of a solution based on evidence, logic, and relevant context. Thirdly, the strategy for strengthening critical thinking in the inference ability indicator is outlined as follows: contextualization learning activities, data and fact exploration, hypothesis formulation, use of evidence and calculations, and reflection and generalization.

7. Suggestion

In the subsequent analysis, it is posited that educational institutions and relevant stakeholders should consider implementing measures to enhance the integration of mathematics education, focusing on deep learning. This integration should be facilitated by providing professional development opportunities for educators, developing curricula that prioritize critical thinking skills as the primary objective, and optimizing technology-based learning tools that support student exploration, evaluation, and inference. The implementation of such a learning approach is expected to foster deep conceptual understanding in students, thereby preparing them to face the contextual challenges characteristic of the 21st century.

Declarations

Author Contributions. (Author 1: Coordinator responsible for the entire research activities, as well as the presentation of progress reports, final research reports, and journal publications. Authors 2, 3, 4, and 5: Coordinated research data exploration and implemented workshops, peer teaching, and FGDs; assisted in the presentation of progress reports, final research reports, and journal publication.

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Data Availability Statement. Information supporting the results of this study is private because it is bound by the ethical provisions governing research on human subjects.

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